ASTOS Model Setup – Sub-Orbital Rocket Trajectory Modelling

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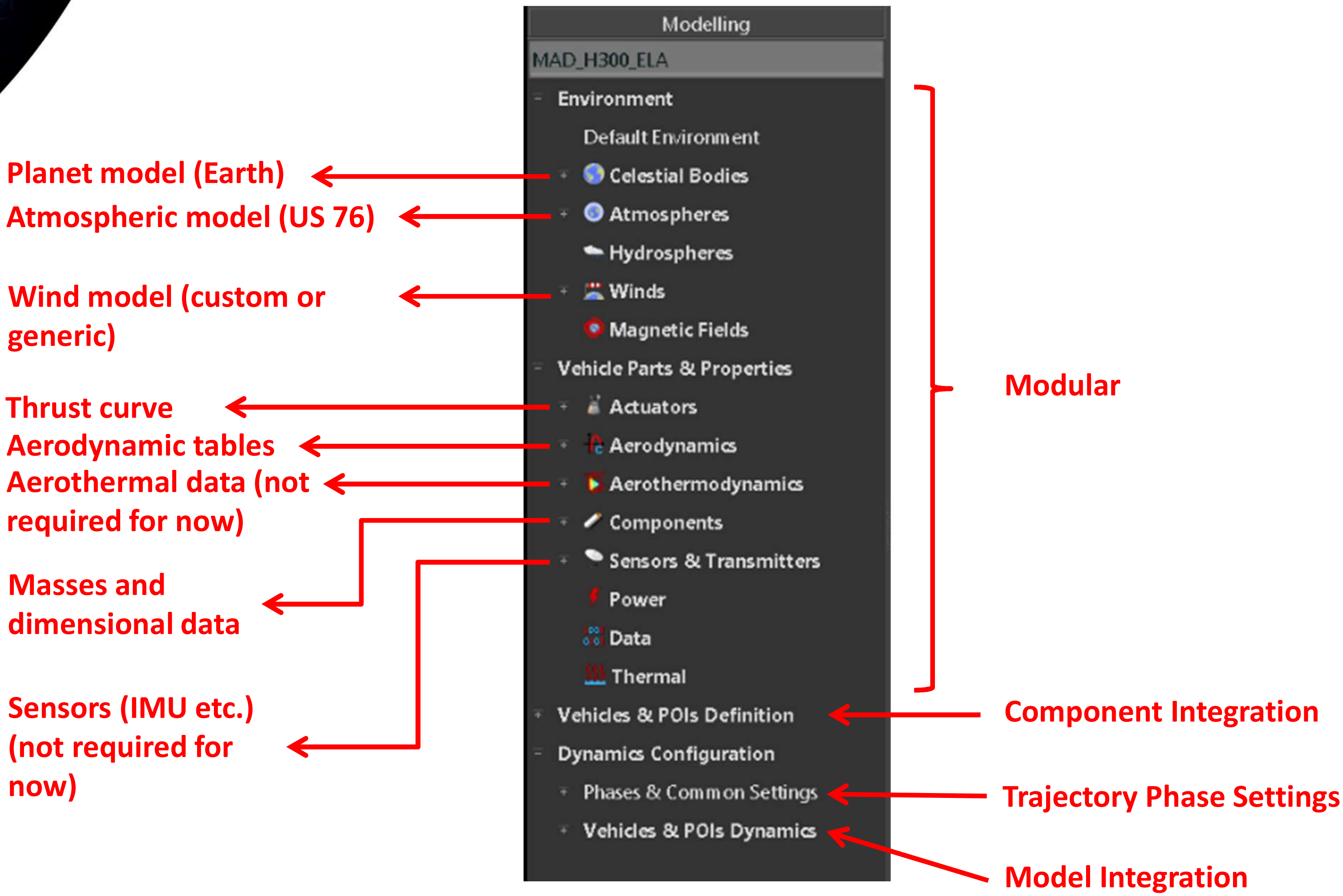
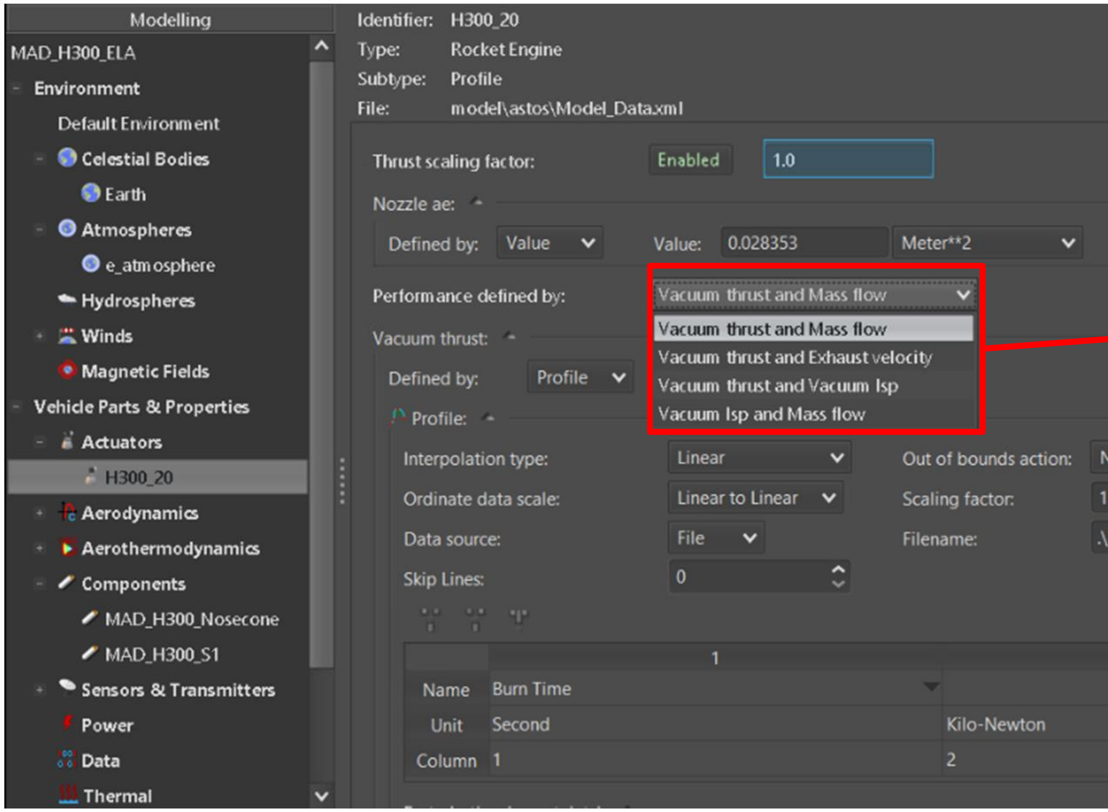
High Level Overview



Thrust Model



Performance definition variations

- **Typically vacuum thrust and mass flow is selected**

- **For simplification, mass flow is usually set as constant value**

Thrust Model

General Thrust Equation

$$F = \dot{m}\,V_e + (P_e - P_0)A_e$$

For vacuum thrust,

$$P_0 = 0\,,$$
$$F_{vac} = \dot{m}\,V_e + P_e\,A_e$$

For sea level thrust,

$$P_0 = P_{SL} = 100\ \mathrm{kPa}$$
$$F_{SL} = \dot{m}\,V_e + (P_e - P_{SL})A_e$$

Conversion of SL thrust to vacuum thrust:

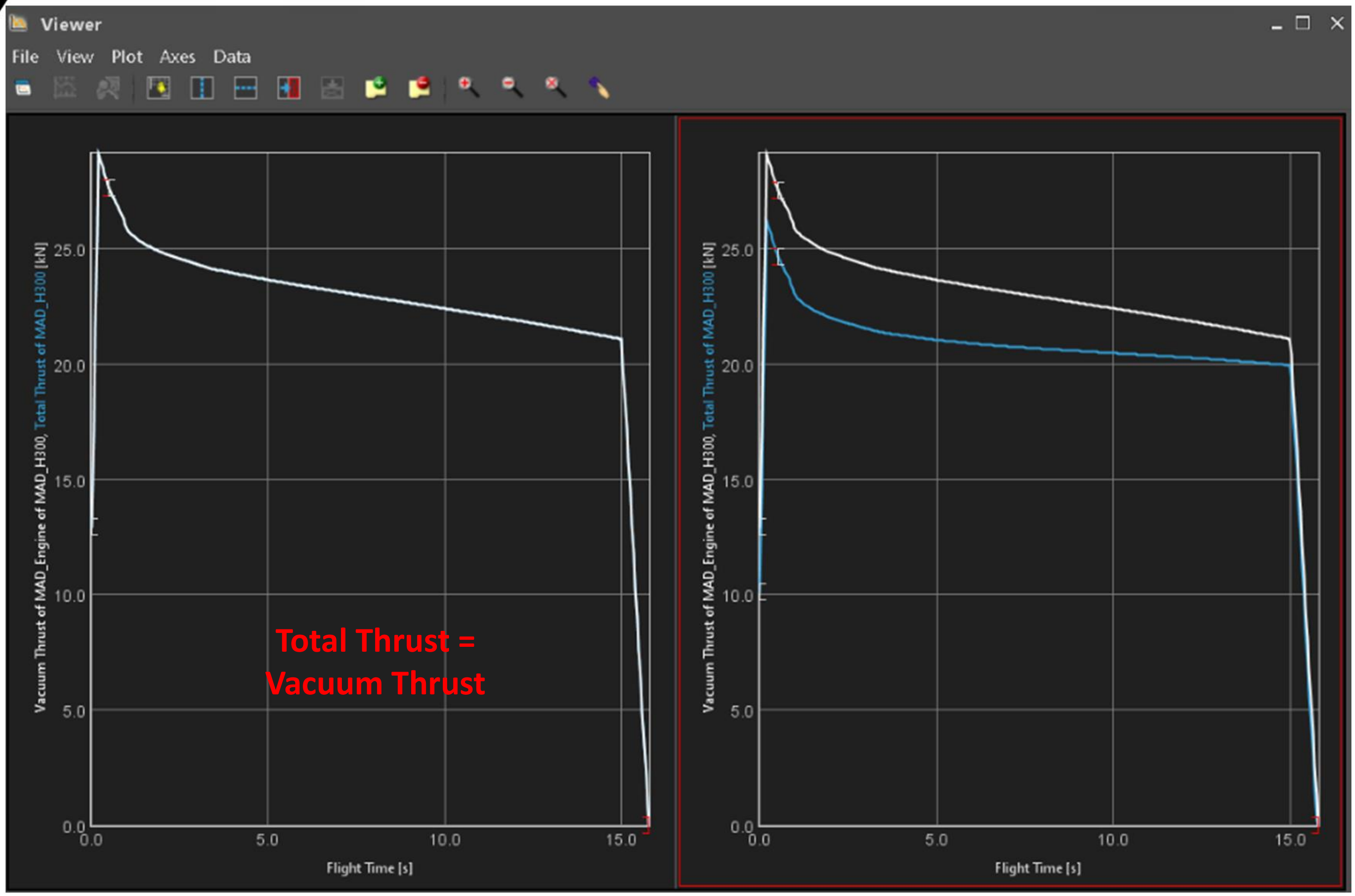
$$F_{vac} = F_{SL} + P_{SL}\,A_e$$

Vacuum thrust curve data as Astos input

Propulsion thrust curve data

Nozzle exhaust area as input

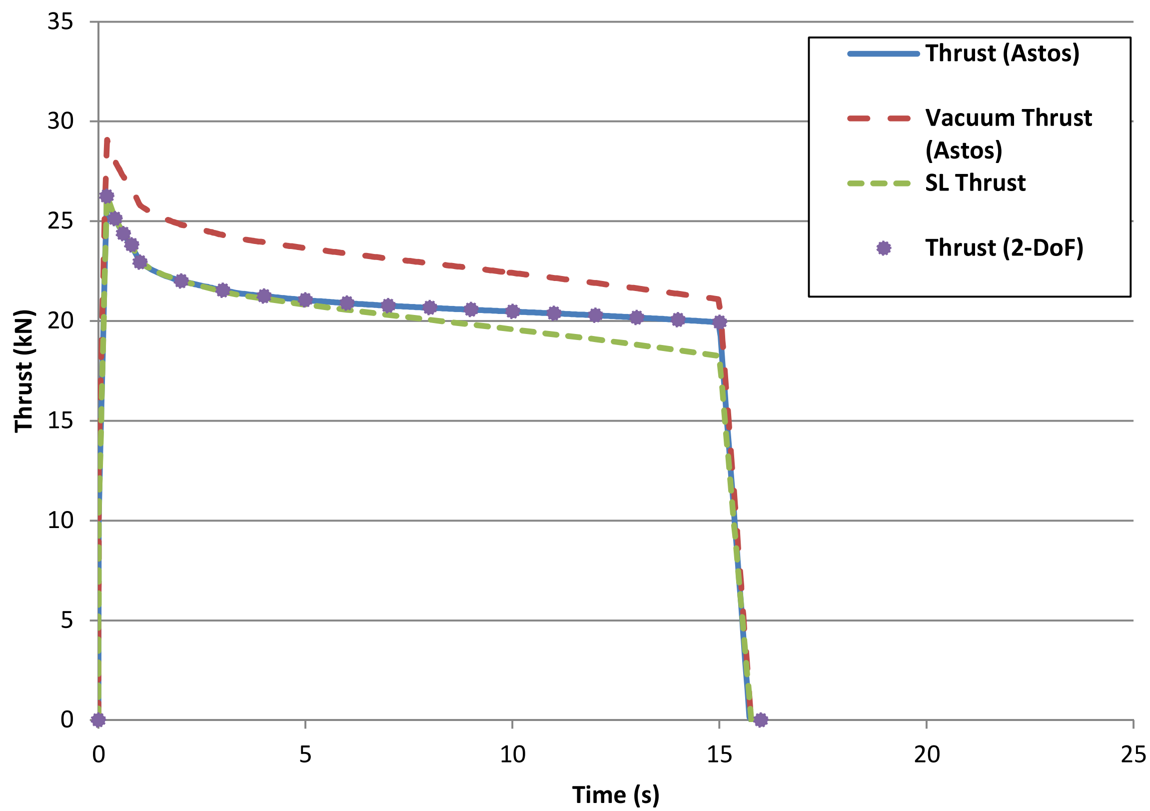
Thrust Model



Nozzle exhaust area = 0

Nozzle exhaust area ≠ 0

Thrust Model



Aerodynamics Model

Force and moment equations in body-fixed coordinates

$$\vec{F}_{\text{aero,B}} = qA_{\text{ref}} \begin{bmatrix} -C_A(x) \\ C_Y(x) + \left(\frac{dC_Y}{d\beta}(x)\right)\beta \\ -C_N(x) - \left(\frac{dC_N}{d\alpha}(x)\right)\alpha \end{bmatrix}_B$$

$$\vec{M}_{\text{aero}} = \begin{bmatrix} l \\ m \\ n \end{bmatrix}_B = qA_{\text{ref}}L_{\text{ref}} \begin{bmatrix} C_{LL_t}(x) \\ C_{M_t}(x) \\ C_{LN_t}(x) \end{bmatrix}_B \begin{matrix} + F_Z\Delta y + F_Y\Delta z \\ + F_Z\Delta x - F_X\Delta z \\ - F_Y\Delta x - F_X\Delta y \end{matrix}$$

Aerodynamics Model

Moment Coefficients Computation (Astos Model Reference)

$$C_{LL_t}(x) = C_{LL}(x) + \beta\left(\frac{dC_{\mathrm{LL}}}{d\beta}(x)\right) + P\left(\frac{dC_{\mathrm{LL}}}{dP}(x)\right) + R\left(\frac{dC_{\mathrm{LL}}}{dR}(x)\right) + \Delta_{fin}\left(\frac{dC_{\mathrm{LL}}}{d\Delta_{fin}}(x)\right)$$

$$C_{M_t}(x) = C_M(x) + \beta\left(\frac{dC_M}{d\beta}(x)\right) + Q\left(\frac{dC_M}{dQ}(x)\right) + \alpha\left(\frac{dC_M}{d\alpha}(x)\right) + \alpha_t\left(\frac{dC_M}{d\alpha_t}(x)\right) + \dot{\alpha}_t\left(\frac{dC_M}{d\dot{\alpha}_t}(x)\right)$$

$$C_{LN_t}(x) = C_{LN}(x) + \beta\left(\frac{dC_{\mathrm{LN}}}{d\beta}(x)\right) + P\left(\frac{dC_{\mathrm{LN}}}{dP}(x)\right) + R\left(\frac{dC_{\mathrm{LN}}}{dR}(x)\right)$$

Currently Used

$$C_{LL_t}(x) = \cancel{C_{LL}(x)} + \cancel{\beta\left(\frac{dC_{\mathrm{LL}}}{d\beta}(x)\right)} + P\left(\frac{dC_{\mathrm{LL}}}{dP}(x)\right) + \cancel{R\left(\frac{dC_{\mathrm{LL}}}{dR}(x)\right)} + \Delta_{fin}\left(\frac{dC_{\mathrm{LL}}}{d\Delta_{fin}}(x)\right)$$

$$C_{M_t}(x) = \cancel{C_M(x)} + \cancel{\beta\left(\frac{dC_M}{d\beta}(x)\right)} + Q\left(\frac{dC_M}{dQ}(x)\right) + \cancel{\alpha\left(\frac{dC_M}{d\alpha}(x)\right)} + \cancel{\alpha_t\left(\frac{dC_M}{d\alpha_t}(x)\right)} + \cancel{\dot{\alpha}_t\left(\frac{dC_M}{d\dot{\alpha}_t}(x)\right)}$$

$$C_{LN_t}(x) = \cancel{C_{LN}(x)} + \cancel{\beta\left(\frac{dC_{\mathrm{LN}}}{d\beta}(x)\right)} + \cancel{P\left(\frac{dC_{\mathrm{LN}}}{dP}(x)\right)} + R\left(\frac{dC_{\mathrm{LN}}}{dR}(x)\right)$$

Aerodynamics Model

For rotational symmetry, we assume:

$$\frac{dC_Y}{d\beta}(x) = -\frac{dC_N}{d\alpha}(x)$$

$$\frac{dC_M}{dQ}(x) = \frac{dC_{\mathrm{LN}}}{dR}(x)$$

Equivalent notations when compared to other literature:

$\frac{dC_N}{d\alpha}(x)$ – C_{N_A}

$\frac{dC_Y}{d\beta}(x)$ – C_{y_β}

$\frac{dC_{\mathrm{LL}}}{dP}(x)$ – C_{L_P}

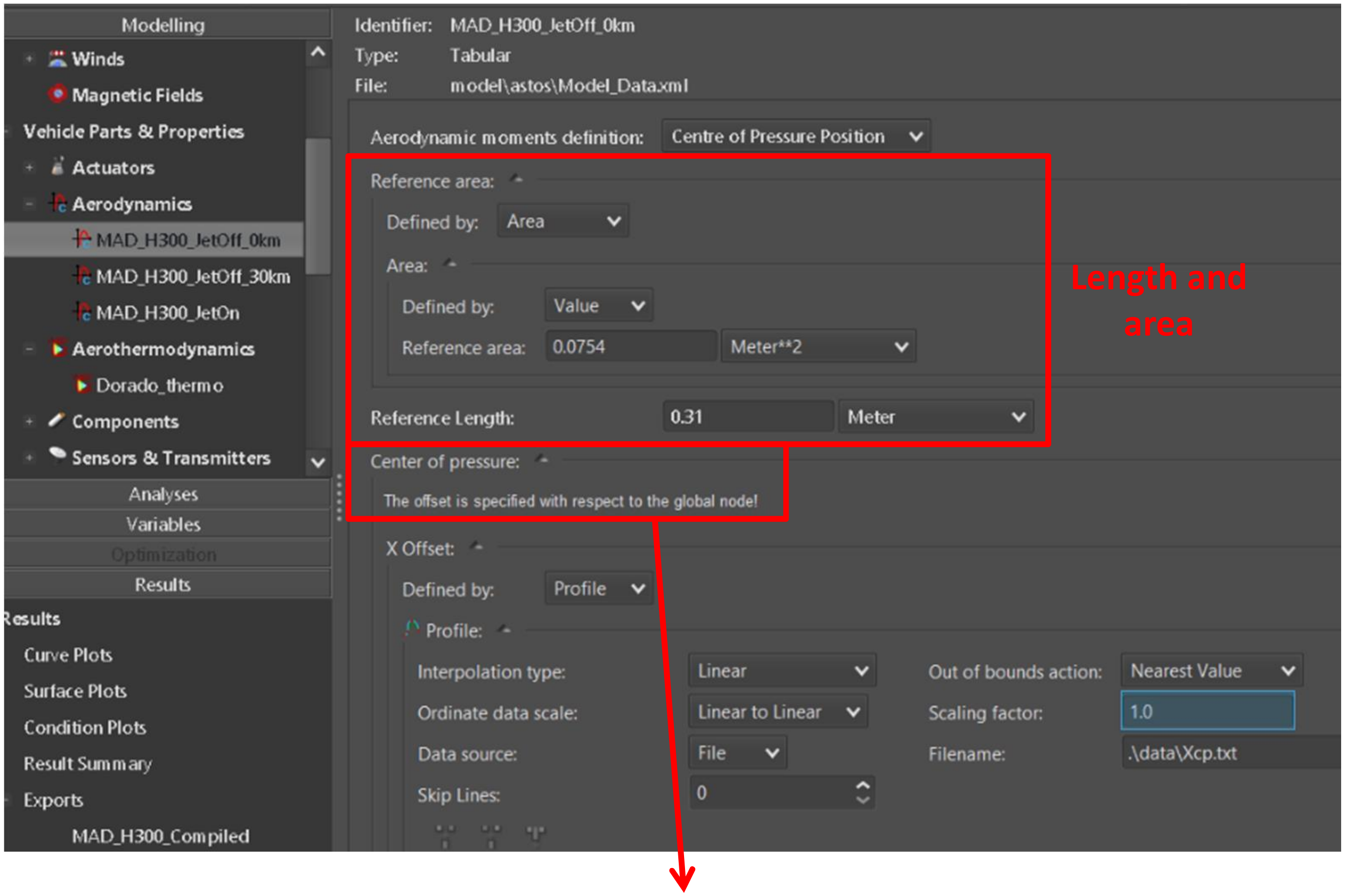
$\frac{dC_{\mathrm{LL}}}{d\Delta fin}(x)$ – C_{L_δ}

$\frac{dC_M}{dQ}(x)$ – C_{M_Q}

$\frac{dC_{\mathrm{LN}}}{dR}(x)$ – C_{N_R}

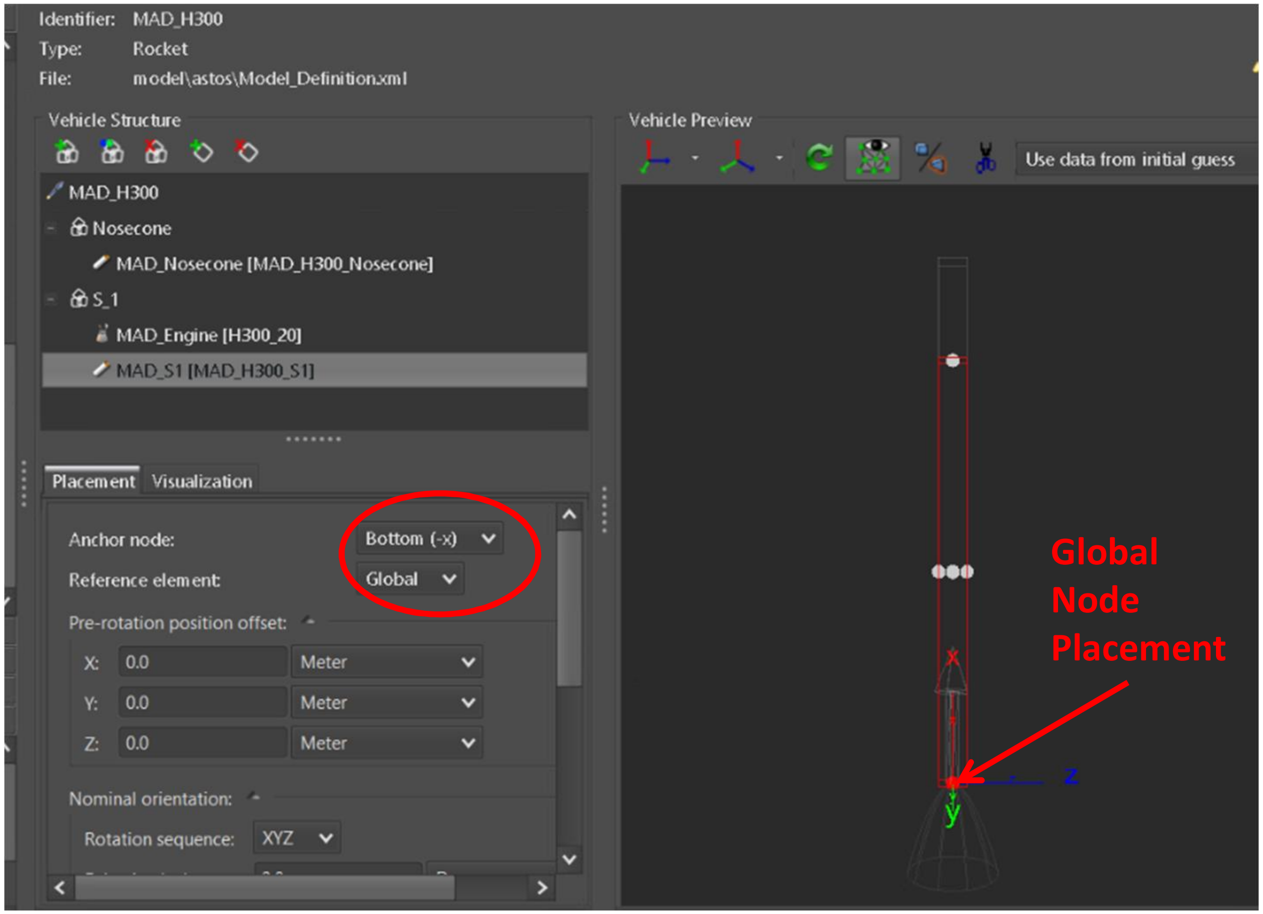
Aerodynamics Model

Definition of Reference Quantities



XCP point is relative to where global node is positioned.

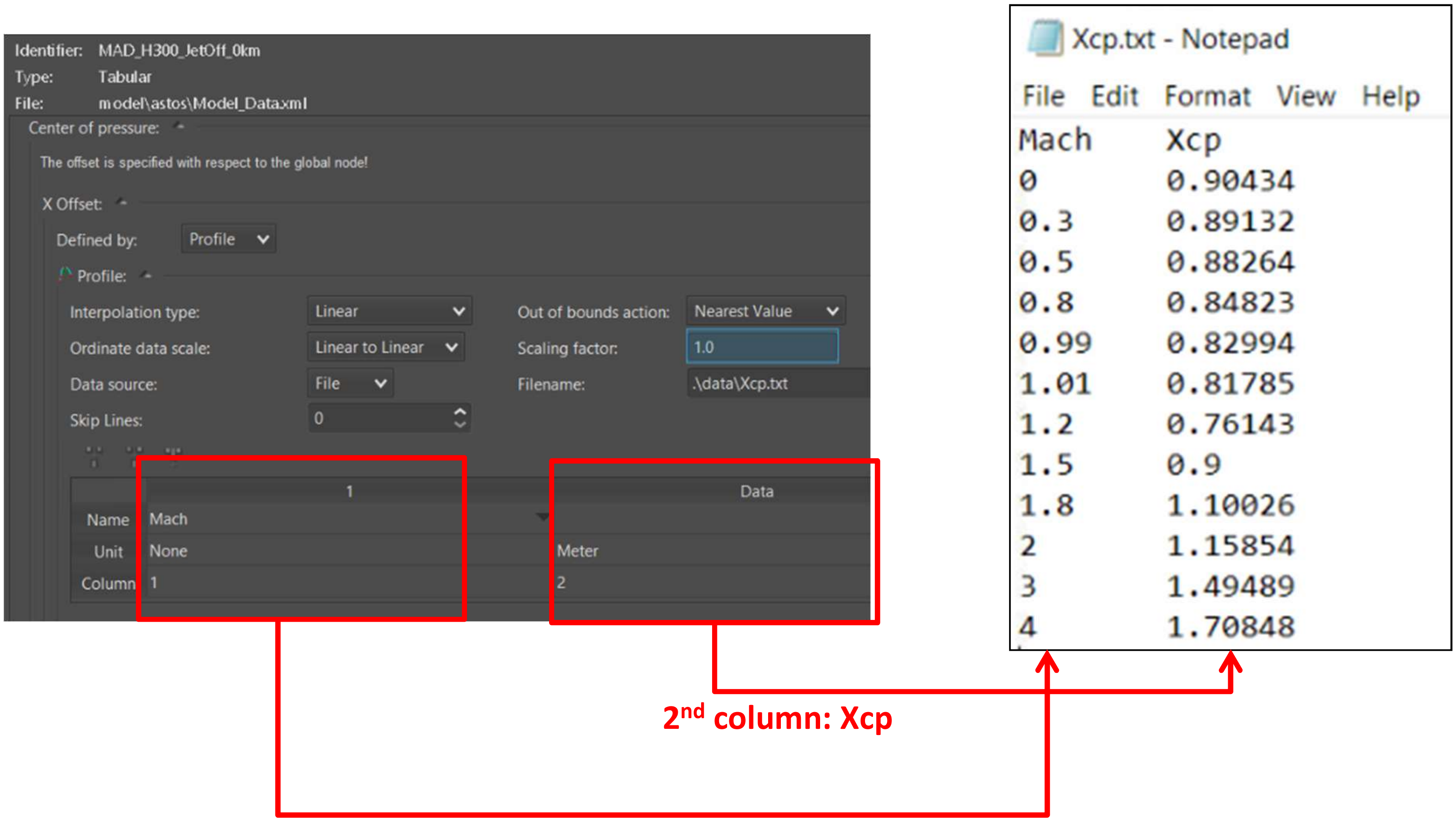
Aerodynamics Model



In this case, XCP is defined +ve since global node is at the bottom. In the case where global node is placed at tip of nose, XCP should be defined –ve.

Aerodynamics Model

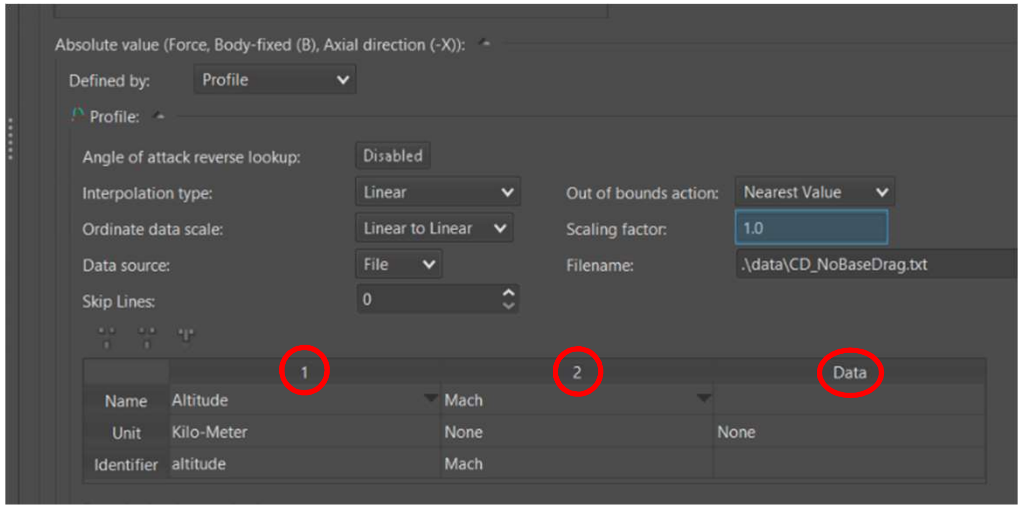
1D Interpolation

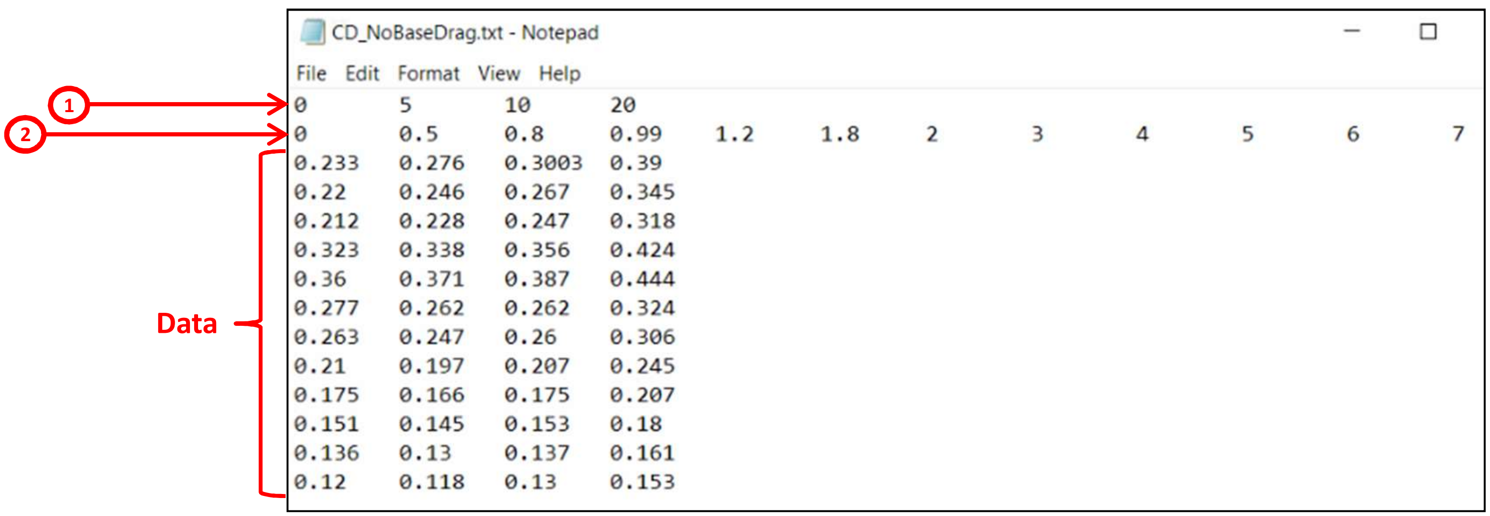


1st column : Mach (Values must be monotonically increasing)

Aerodynamics Model

2D Interpolation





0	5	10	20								
0	0.5	0.8	0.99	1.2	1.8	2	3	4	5	6	7
0.233	0.276	0.3003	0.39								
0.22	0.246	0.267	0.345								
0.212	0.228	0.247	0.318								
0.323	0.338	0.356	0.424								
0.36	0.371	0.387	0.444								
0.277	0.262	0.262	0.324								
0.263	0.247	0.26	0.306								
0.21	0.197	0.207	0.245								
0.175	0.166	0.175	0.207								
0.151	0.145	0.153	0.18								
0.136	0.13	0.137	0.161								
0.12	0.118	0.13	0.153								

Aerodynamics Model

2D Interpolation (according to Astos)

Example 1: 4 x 5 data matrix

The tabular data

y \ x	0	0.50	1.00	1.30	1.50
1.00	0.11	0.12	0.13	0.14	0.15
2.00	0.21	0.22	0.23	0.24	0.25
3.00	0.31	0.32	0.33	0.34	0.35
4.00	0.41	0.42	0.43	0.44	0.45

Fig. 10.2: A 4x5 matrix.

should be formatted in a file as follows:

```
0.00 0.50 1.00 1.30 1.50      - 1st indep. variable x
1.00 2.00 3.00 4.00           - 2nd indep. variable y
0.11 0.12 0.13 0.14 0.15     \
0.21 0.22 0.23 0.24 0.25      | x-y matrix
0.31 0.32 0.33 0.34 0.35      |
0.41 0.42 0.43 0.44 0.45     /
```

Fig. 10.3: A 4x5 matrix format in a file ready to be imported

Aerodynamics Model

Coefficients

Coefficients:

Type	Frame	Axis	Character
Force	Body-fixed (B)	Axial direction (-...	Absolute value
Force	Body-fixed (B)	Lateral direction ...	Sideslip slope
Force	Body-fixed (B)	Normal direction...	Angle of attack s...
Moment	Body-fixed (B)	Roll moment (+X)	Aerodynamic sur...
Moment	Body-fixed (B)	Roll moment (+X)	Body roll rate slo...
Moment	Body-fixed (B)	Pitch moment (+...	Body pitch rate s...
Moment	Body-fixed (B)	Yaw moment (+Z)	Body yaw rate sl...

C_D

C_{y_β}

C_{N_A}

C_{L_P}

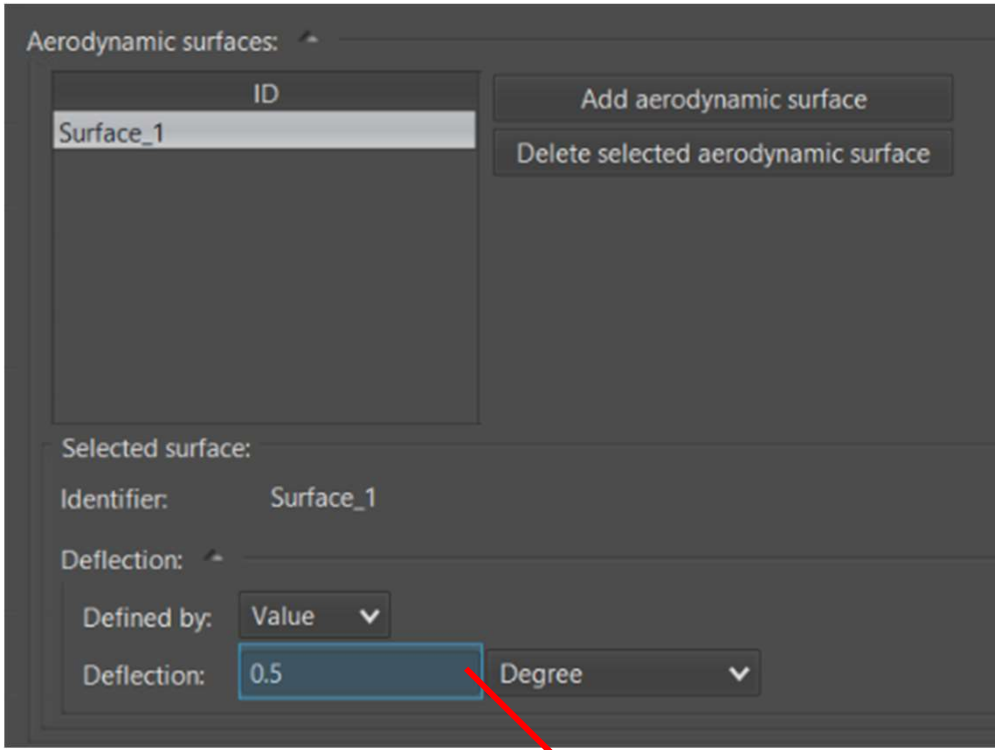
C_{L_δ}

C_{M_Q}

C_{N_R}

Aerodynamics Model

Fin Cant

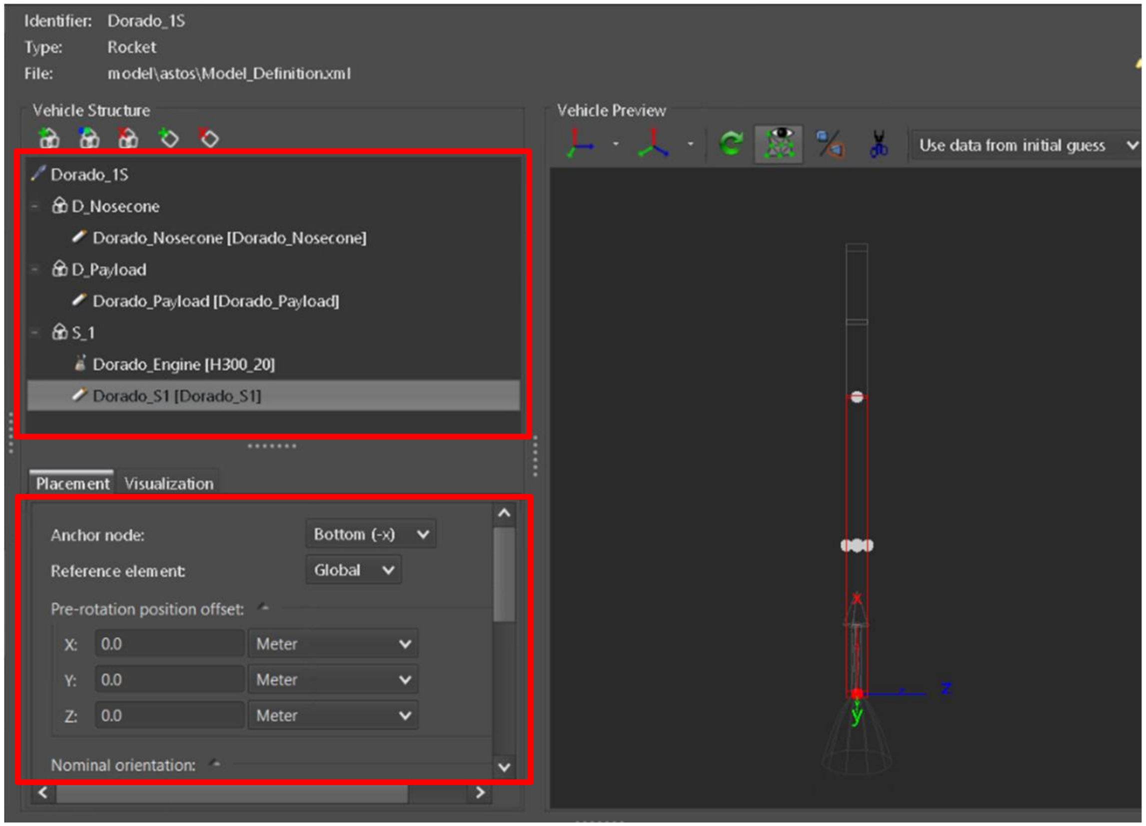


Define fin cant angle

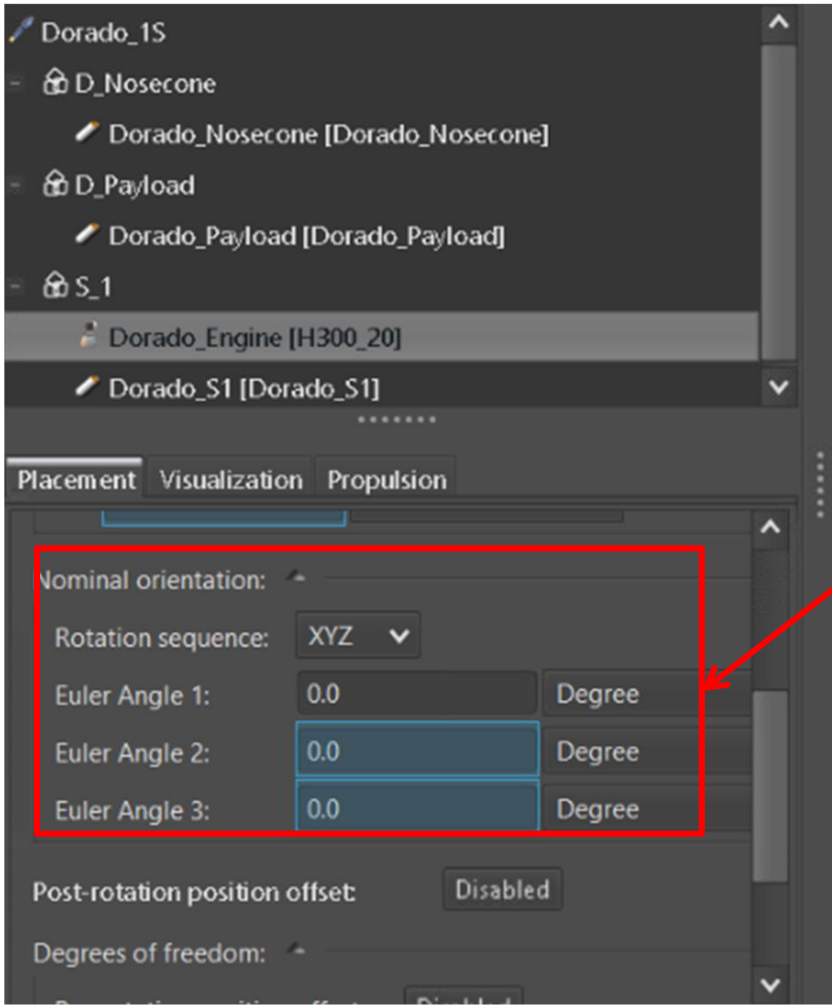
Vehicle Definition

Specify components in use

Define component position using anchor node placement, and position/rotation offsets



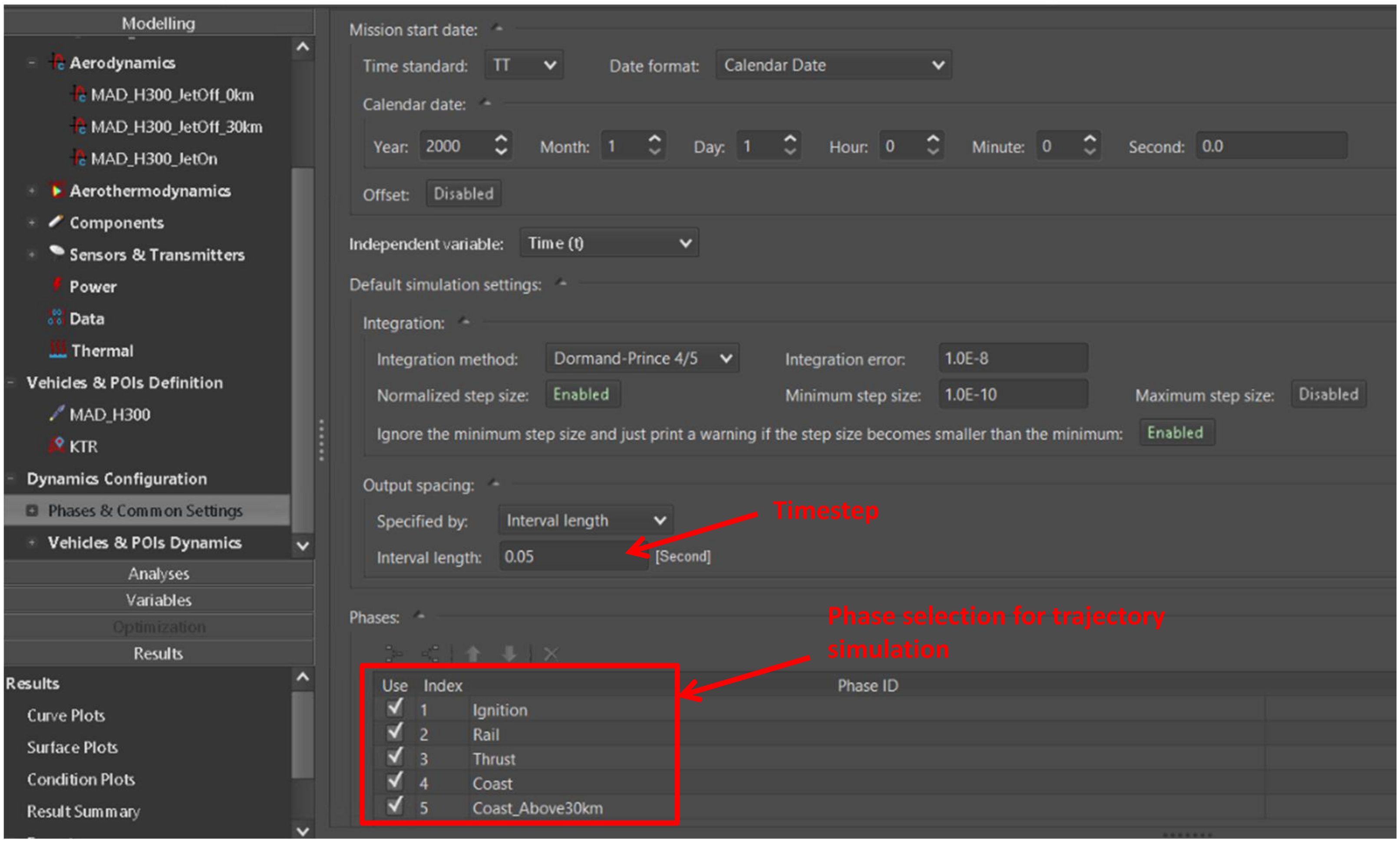
Vehicle Definition



For specifying thrust misalignments during Monte Carlo simulations

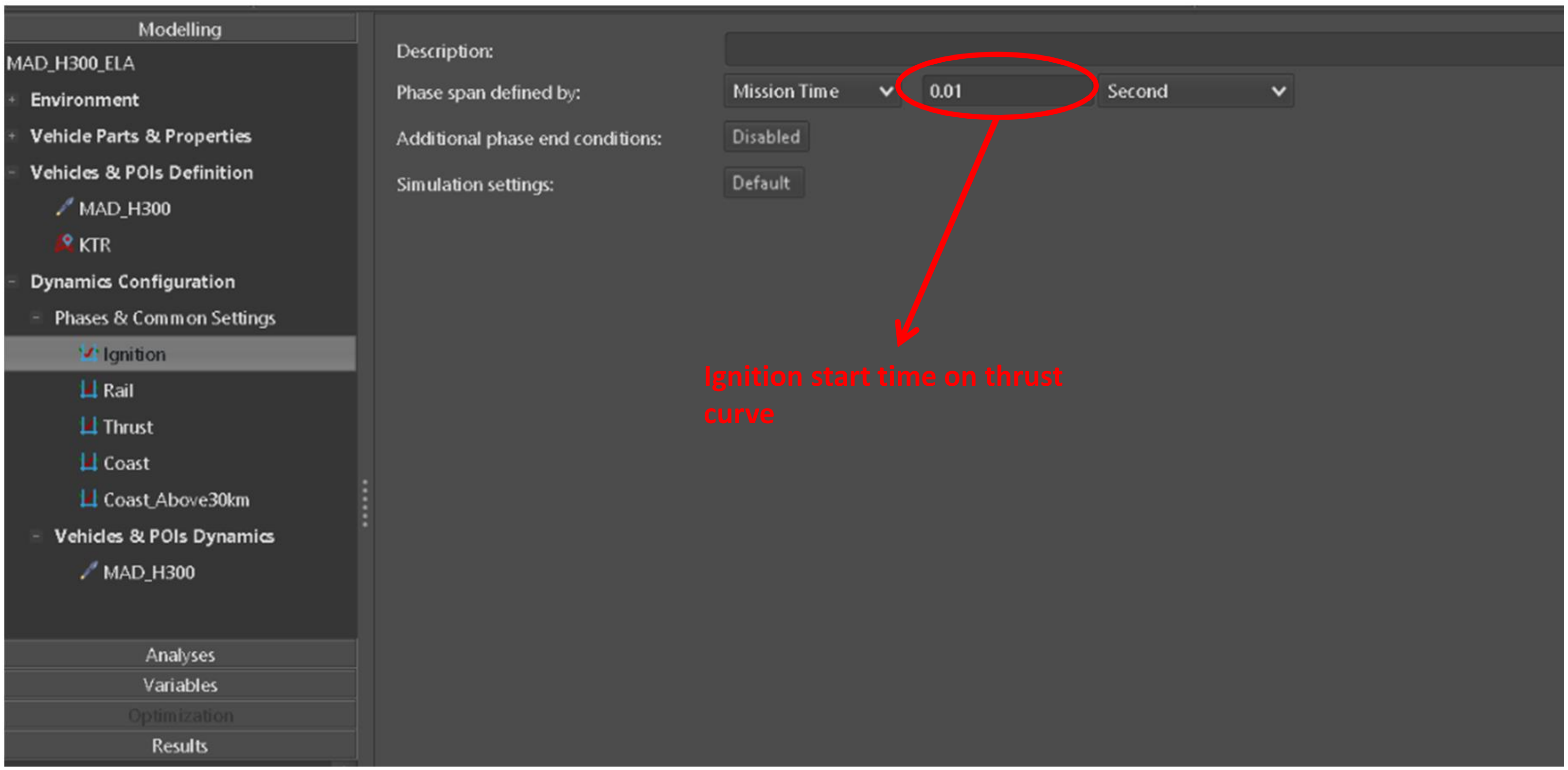
Phase Definition

Overall



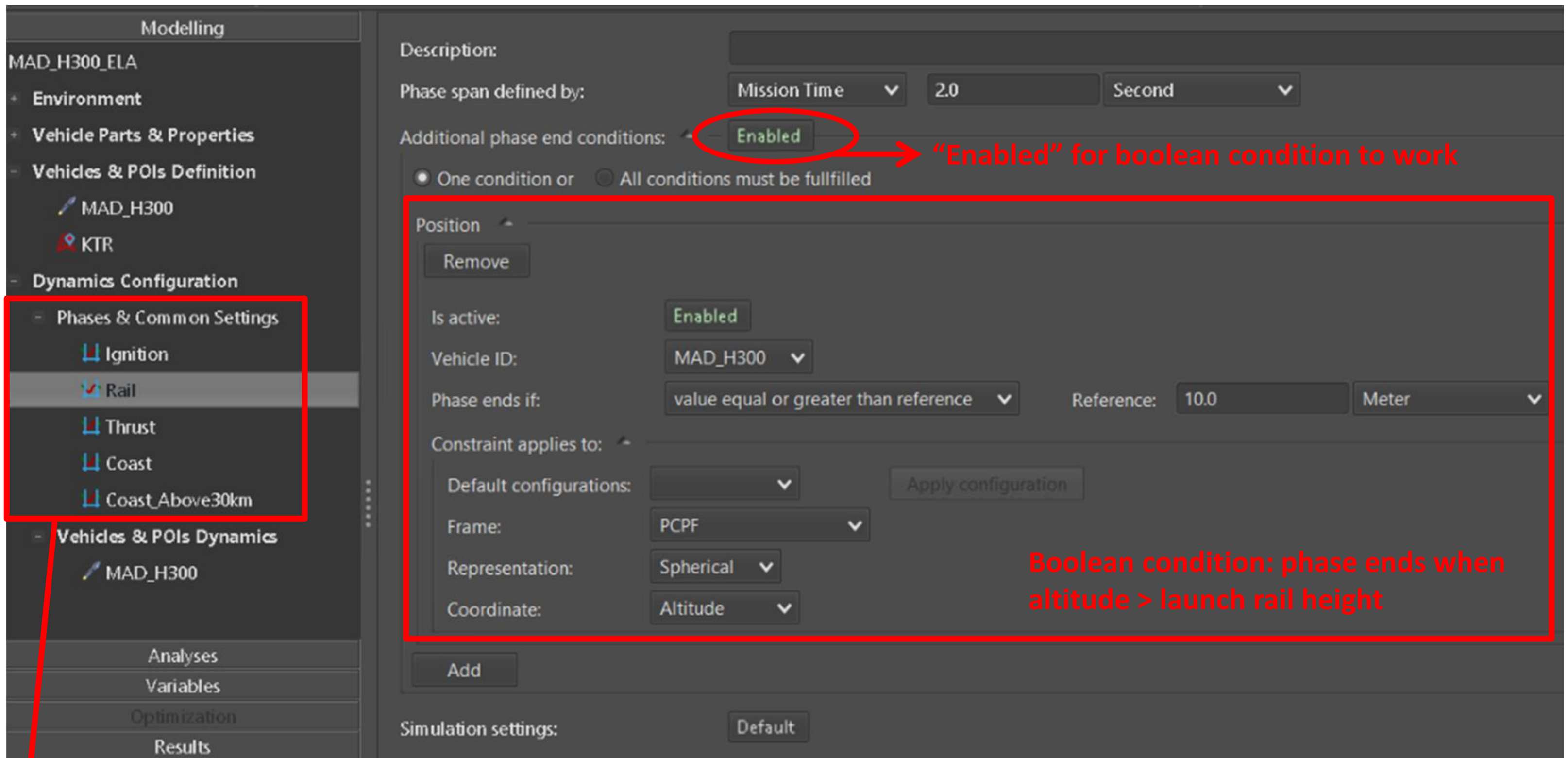
Phase Definition

Ignition Phase



Phase Definition

Rail Phase



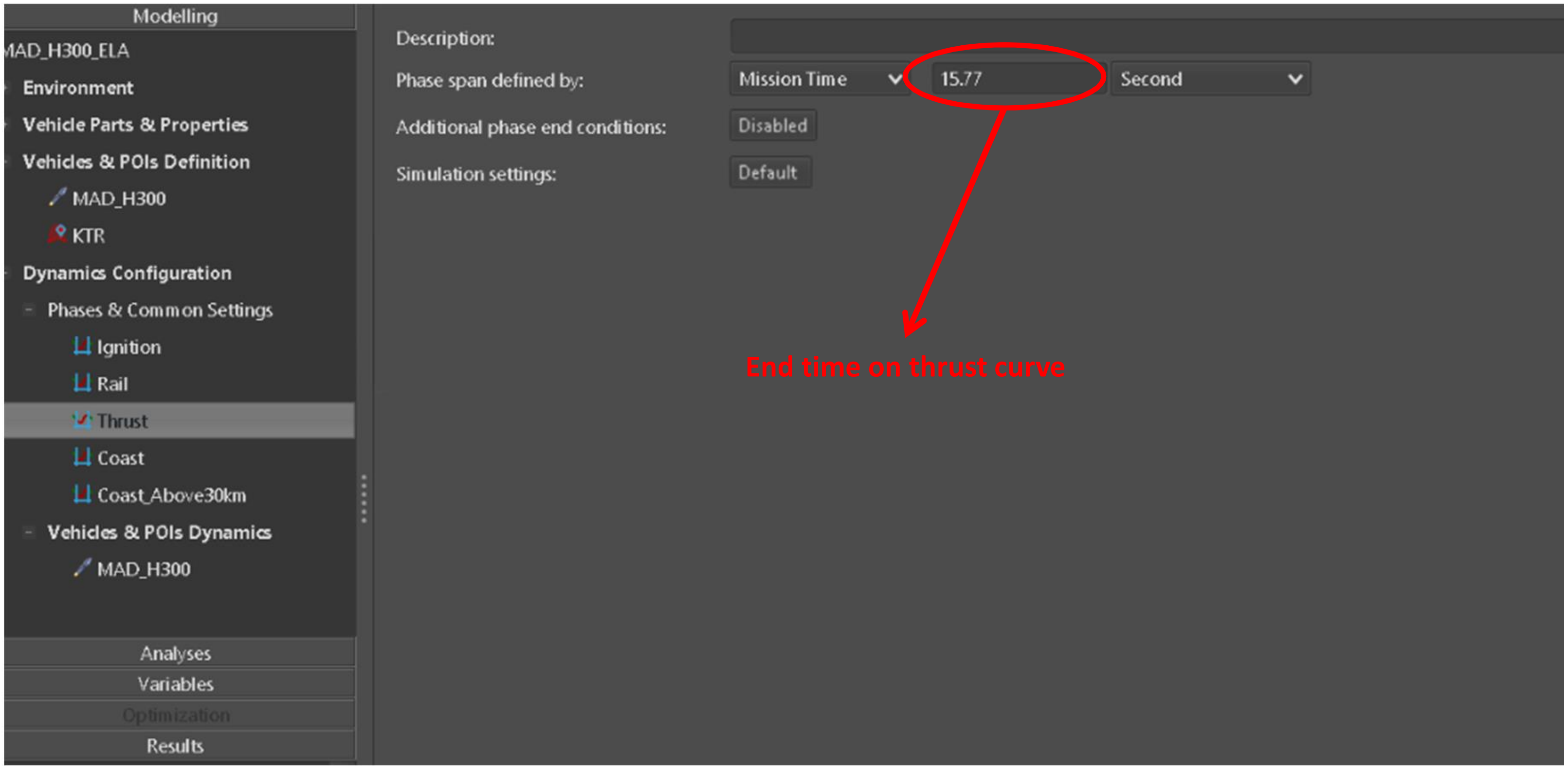
No. of phases vary from case to case, depends on dynamics of trajectory phases

Phase ends when

- Boolean condition is reached, OR
- Phase span reaches mission time or phase time defined

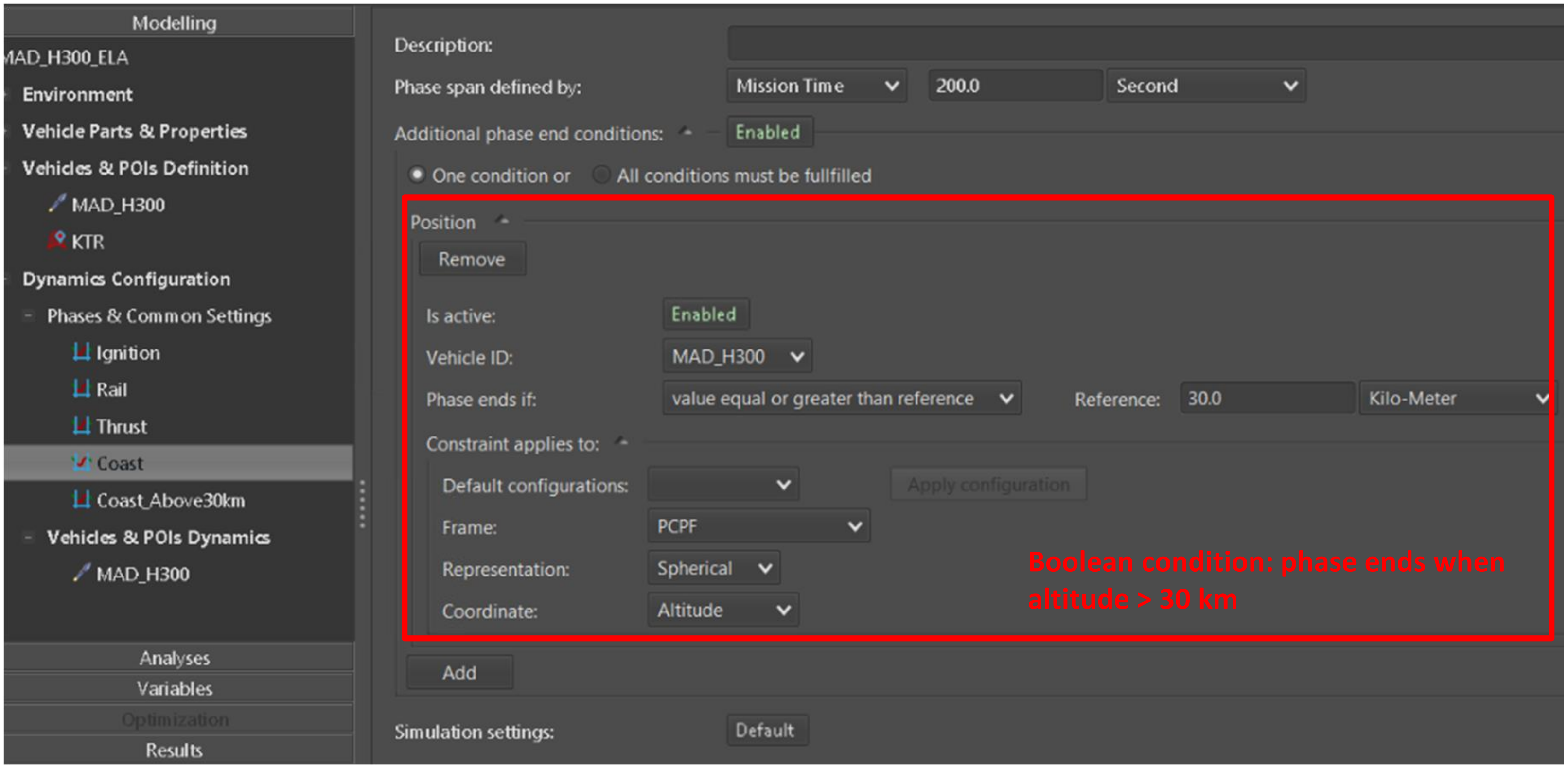
Phase Definition

Thrust Phase



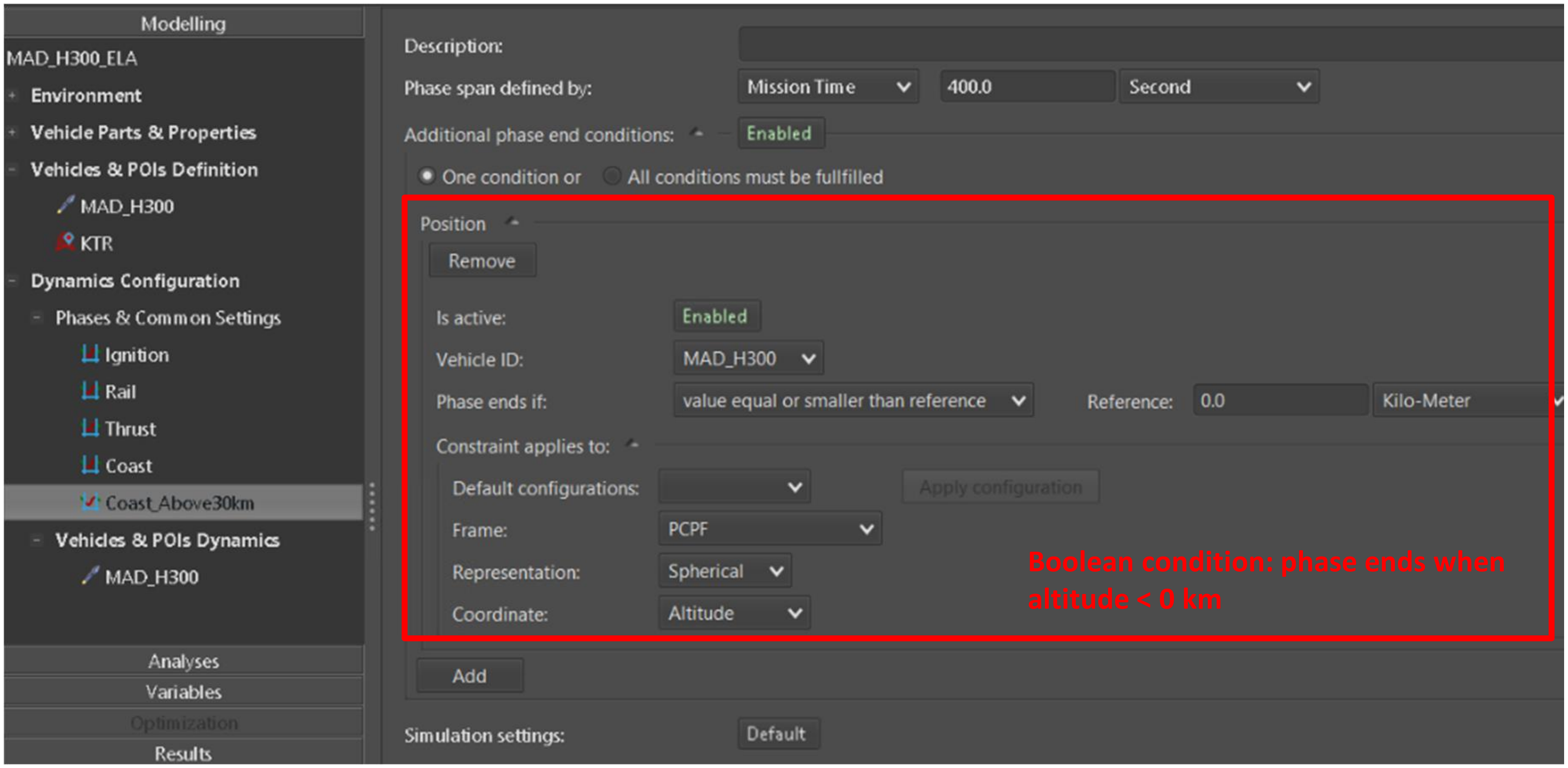
Phase Definition

Coast Phase



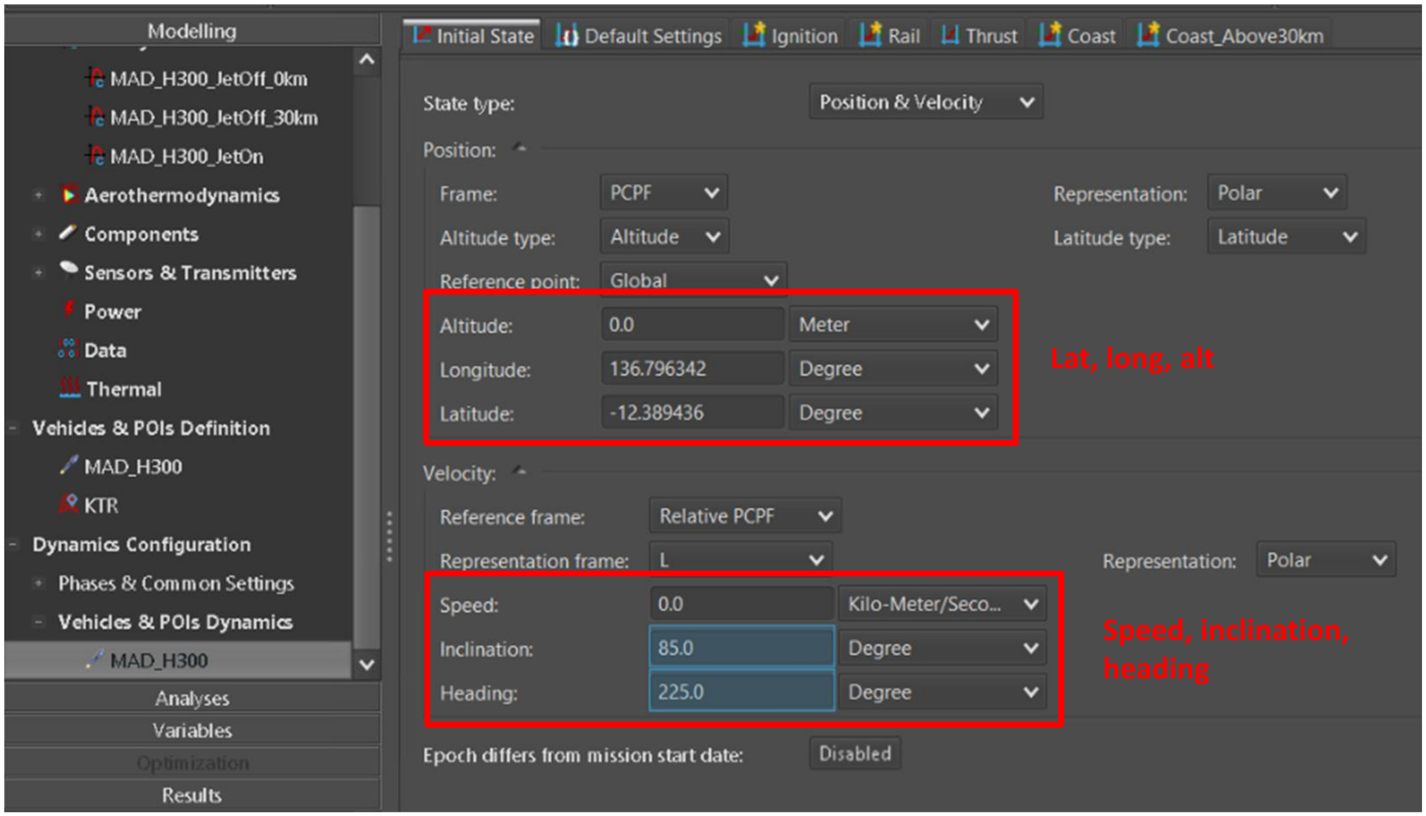
Phase Definition

2nd Coast Phase



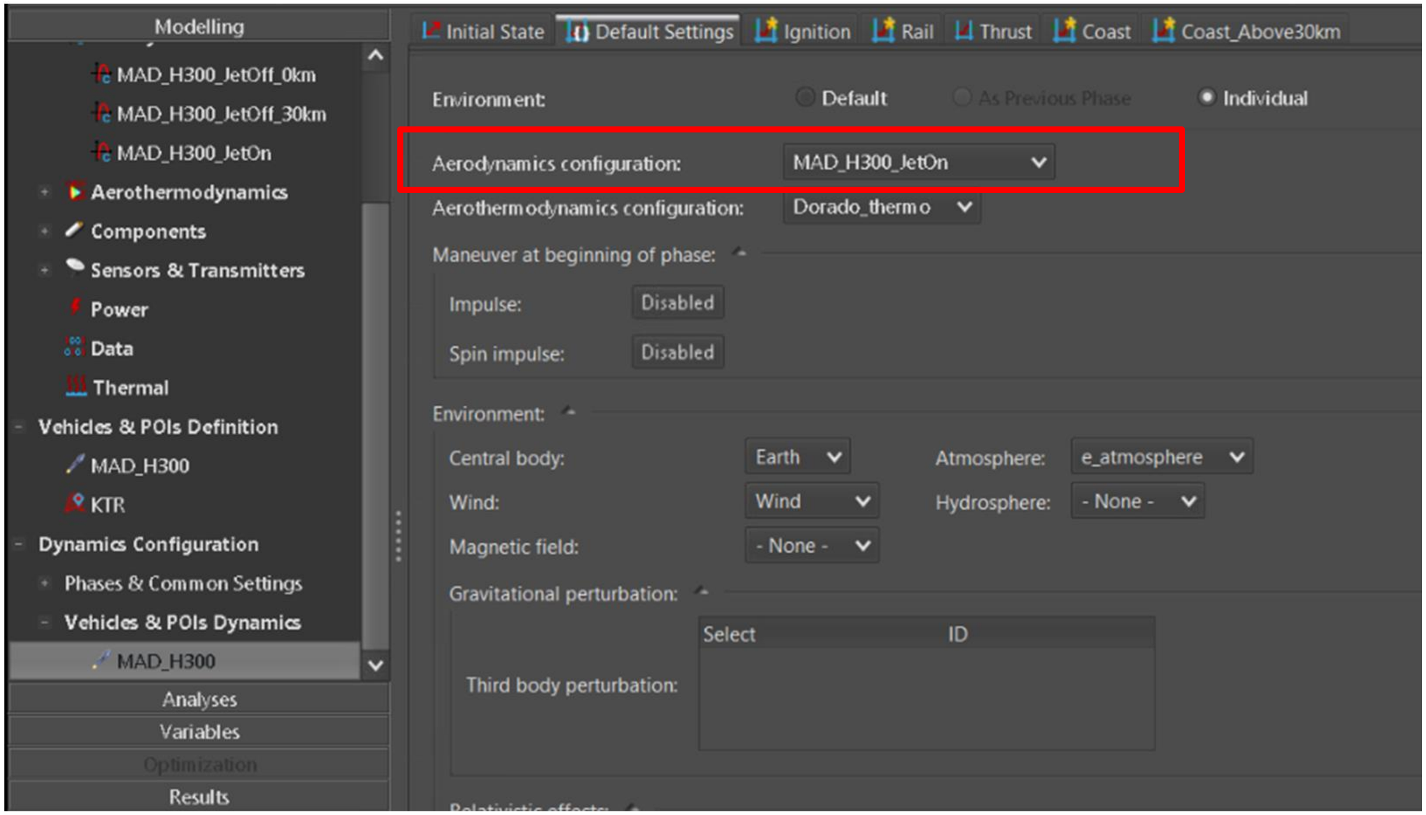
Trajectory Parameters

Initial State



Trajectory Parameters

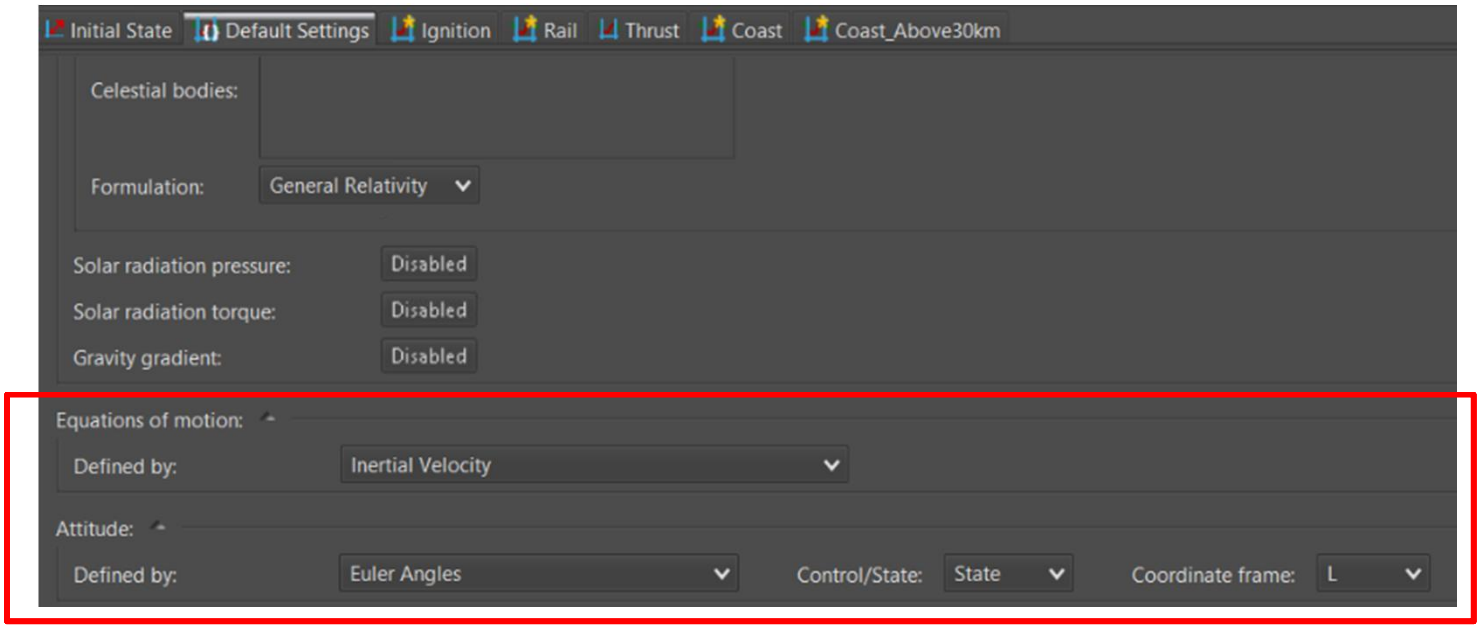
Default Settings



Default aerodynamics model: No base drag

Trajectory Parameters

Default Settings



Default equation of motion and attitude control laws used

Trajectory Parameters

Equations of Motion: Inertial Velocity(Astos Model Reference)

Background

The states V_R, V_λ and V_δ specify the Cartesian components of the inertial velocity vector

$$\hat{V} \equiv \begin{bmatrix} V_R \\ V_\lambda \\ V_\delta \end{bmatrix}_L \tag{4.60}$$

The kinematic state equations represent the kinematic relationship established by the definition of the position and the velocity states

$$\frac{d}{dt}\begin{bmatrix} R \\ \lambda \\ \delta \end{bmatrix} = \begin{bmatrix} V_R \\ \frac{V_\lambda}{R\cos\delta} - \Omega_E \\ \frac{V_\delta}{R} \end{bmatrix} \tag{4.61}$$

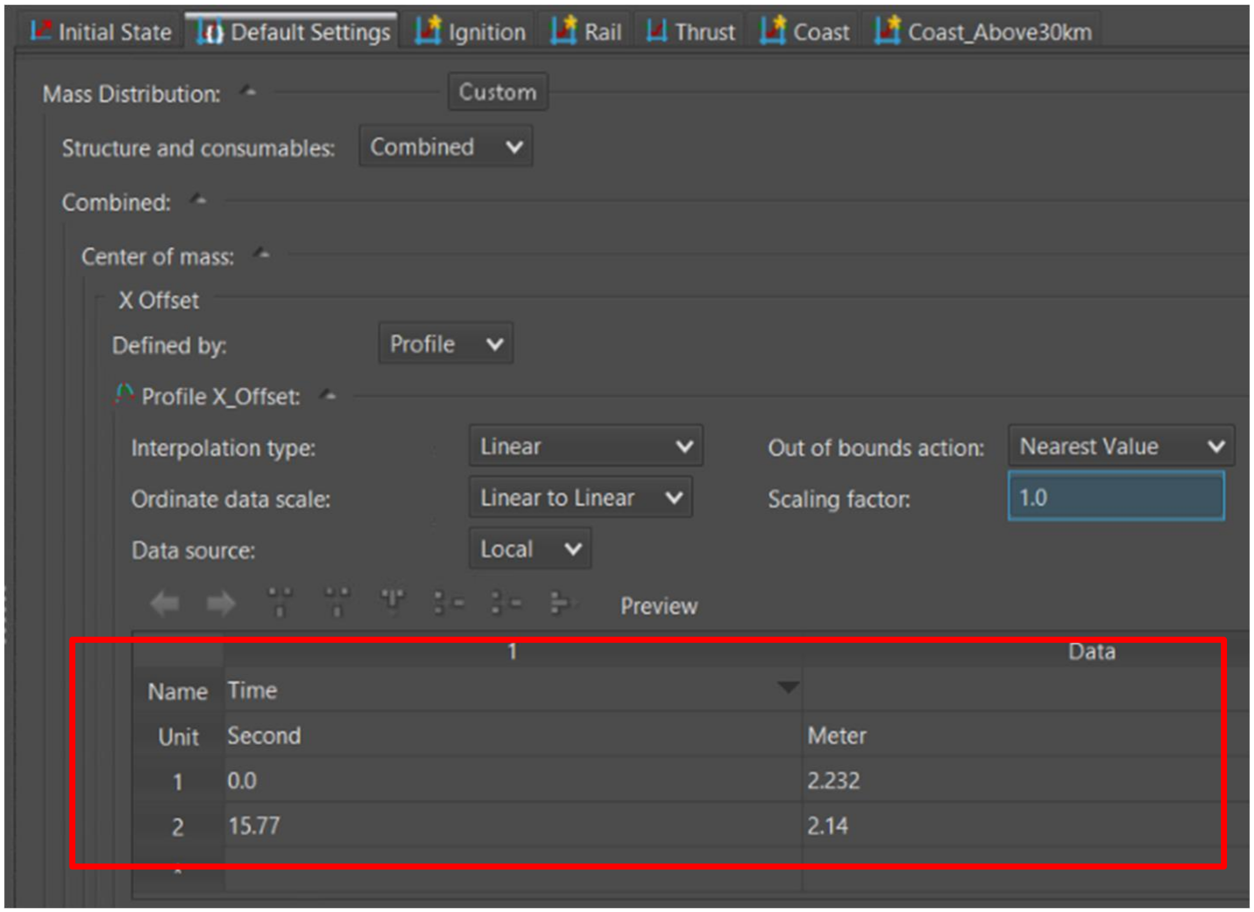
and the dynamic state equations are

$$\frac{d}{dt}\begin{bmatrix} V_R \\ V_\lambda \\ V_\delta \end{bmatrix}_L = \begin{bmatrix} \frac{1}{R} \cdot \left(V_\lambda^2 + V_\delta^2 \right) + \frac{F_R}{m} \\ \frac{1}{R} \cdot V_\lambda \cdot \left(V_\delta \cdot \tan\delta - V_R \right) + \frac{F_\lambda}{m} \\ -\frac{1}{R} \cdot \left(V_\lambda^2 \cdot \tan\delta + V_\delta \cdot V_R \right) + \frac{F_\delta}{m} \end{bmatrix} \tag{4.62}$$

Input to the system of Eq. 4.62 is the acceleration vector acting on the vehicle resulting from gravity, aerodynamic forces, thrust or other perturbations.

Trajectory Parameters

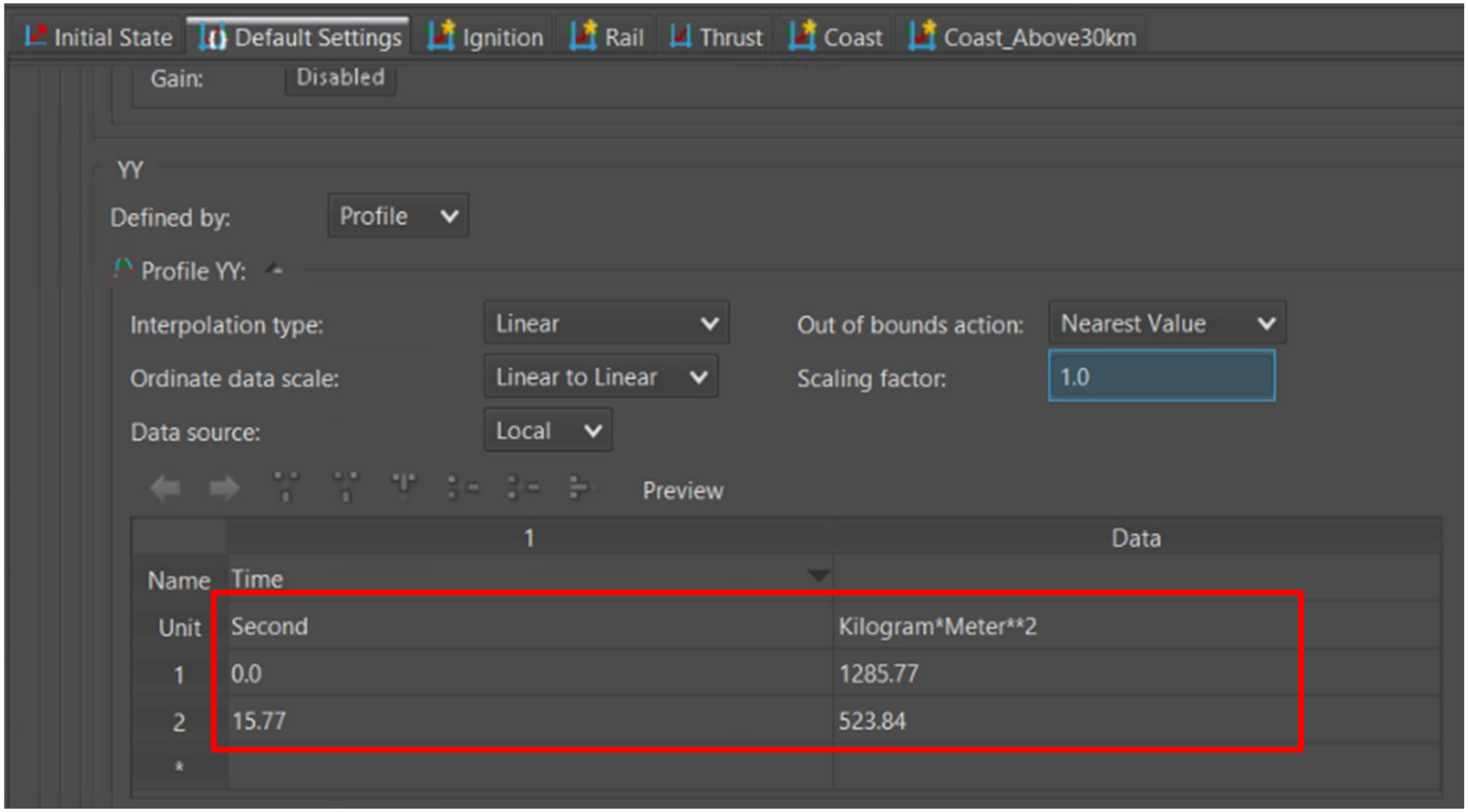
Default Settings



Default CoM

Trajectory Parameters

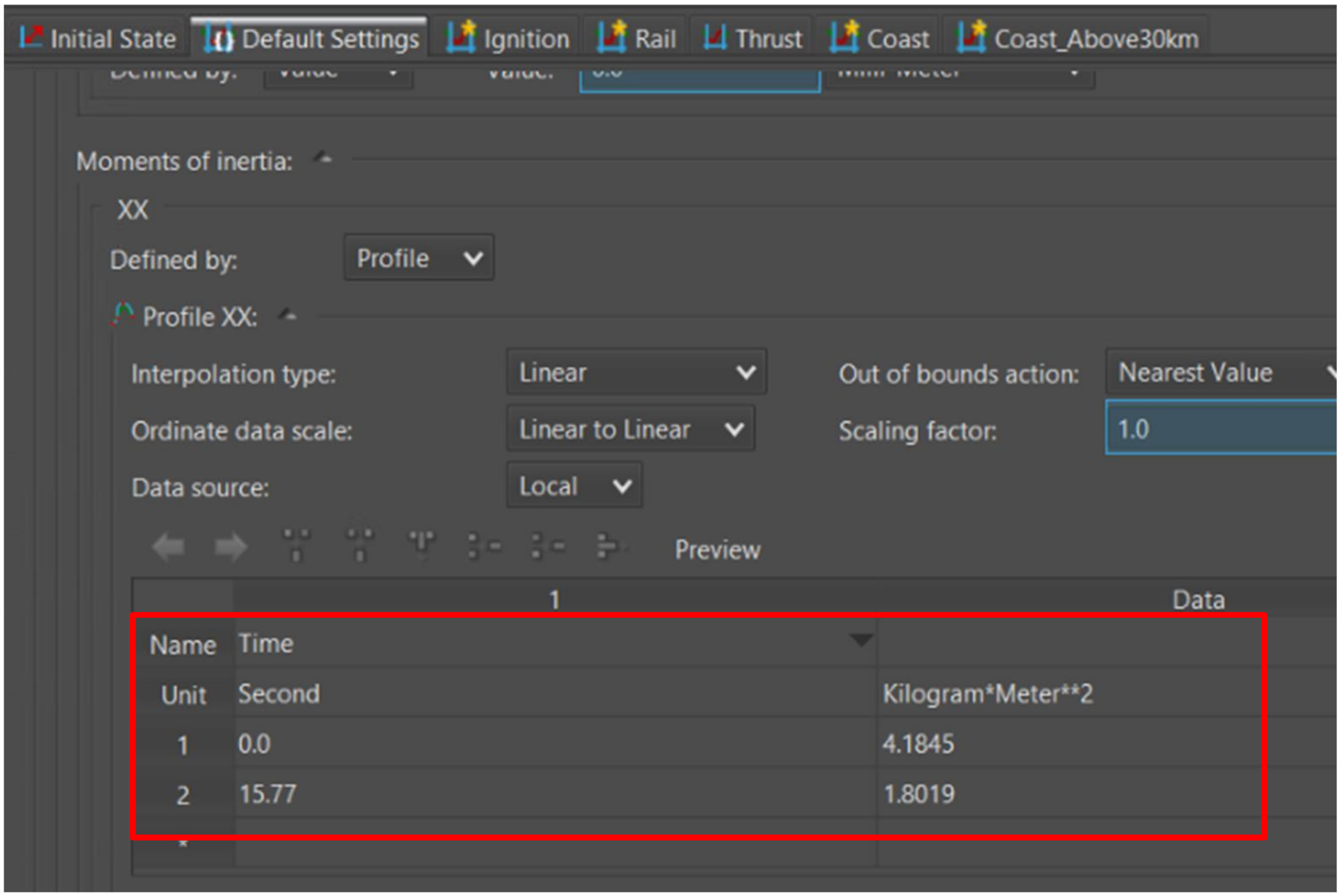
Default Settings



Default MoI (YY) (Same for ZZ)

Trajectory Parameters

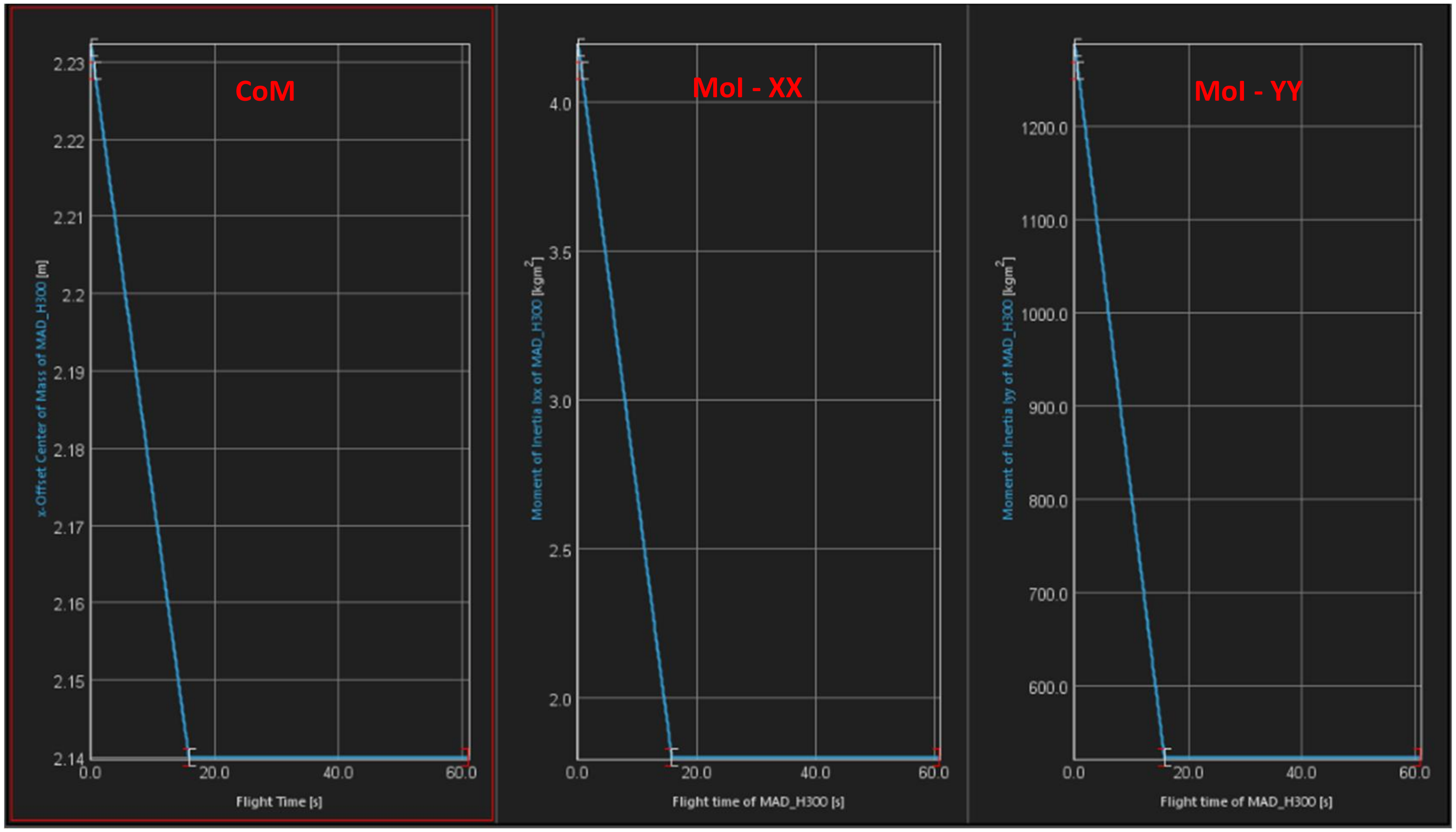
Default Settings



Default MoI (XX)

Trajectory Parameters

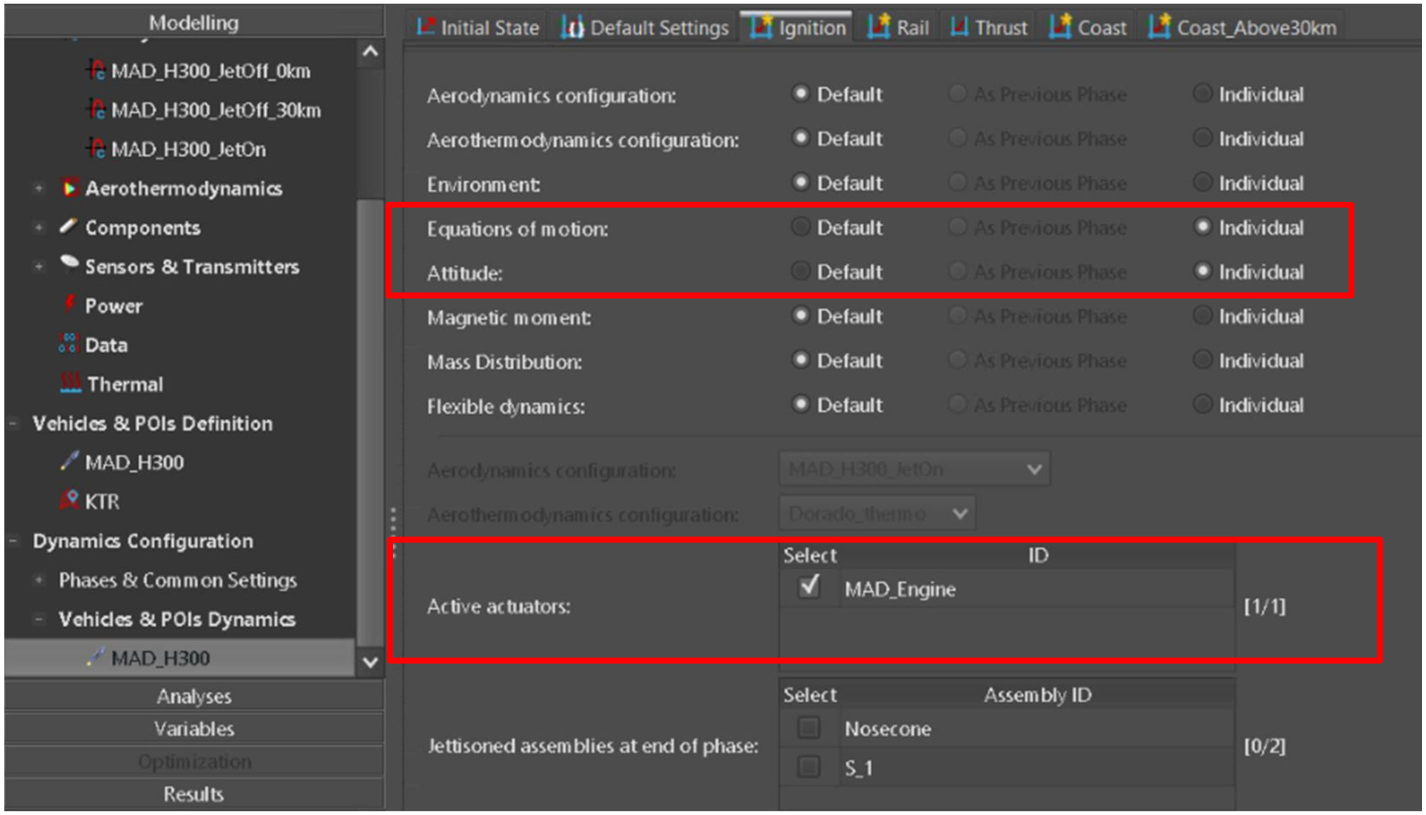
Default Settings



Verification of default settings

Trajectory Parameters

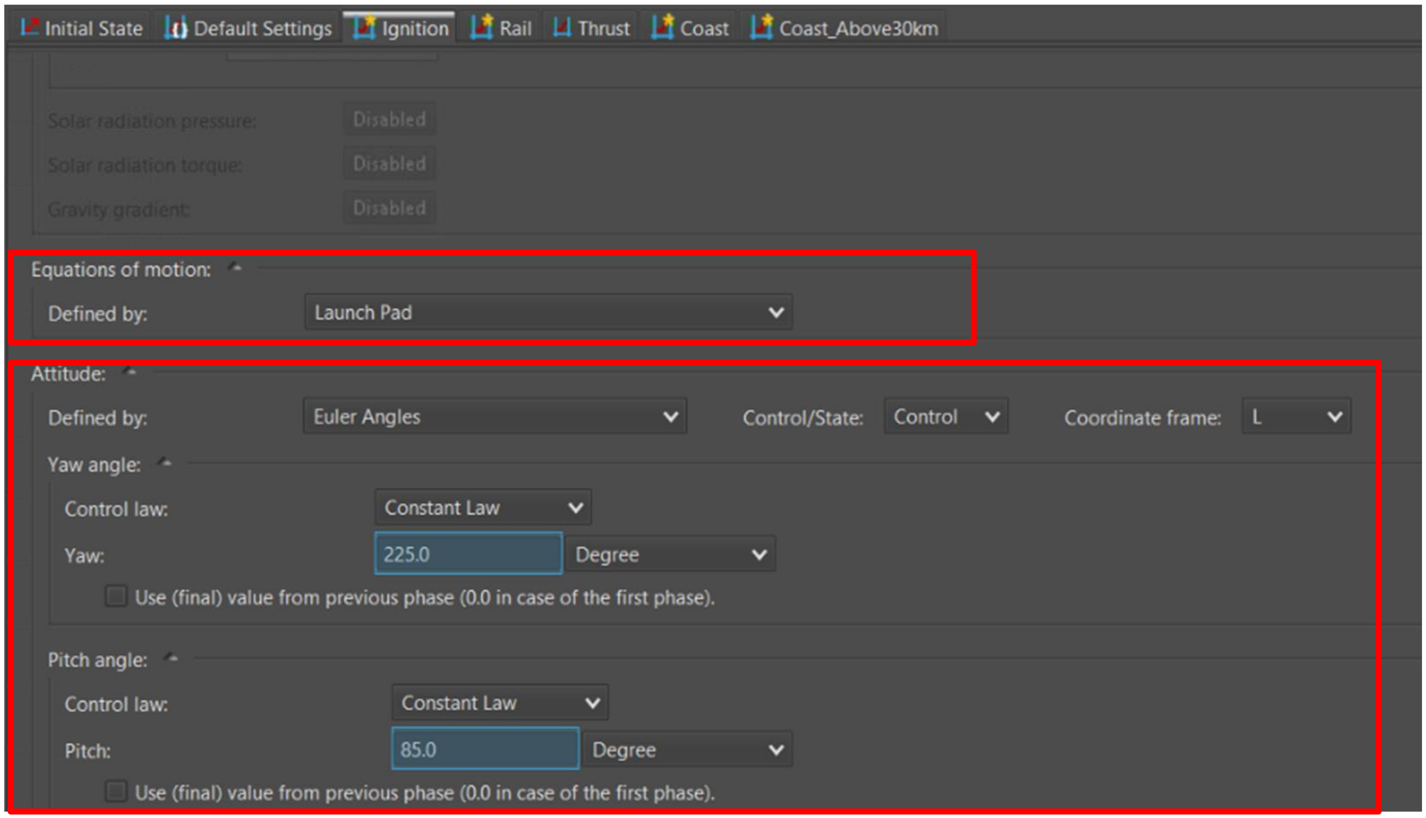
Dynamics - Ignition



Active motor

Trajectory Parameters

Dynamics - Ignition



Trajectory Parameters

Equations of Motion: Launch Pad (Astos Model Reference)

Table 4.2: Definition of flight path velocity state variables

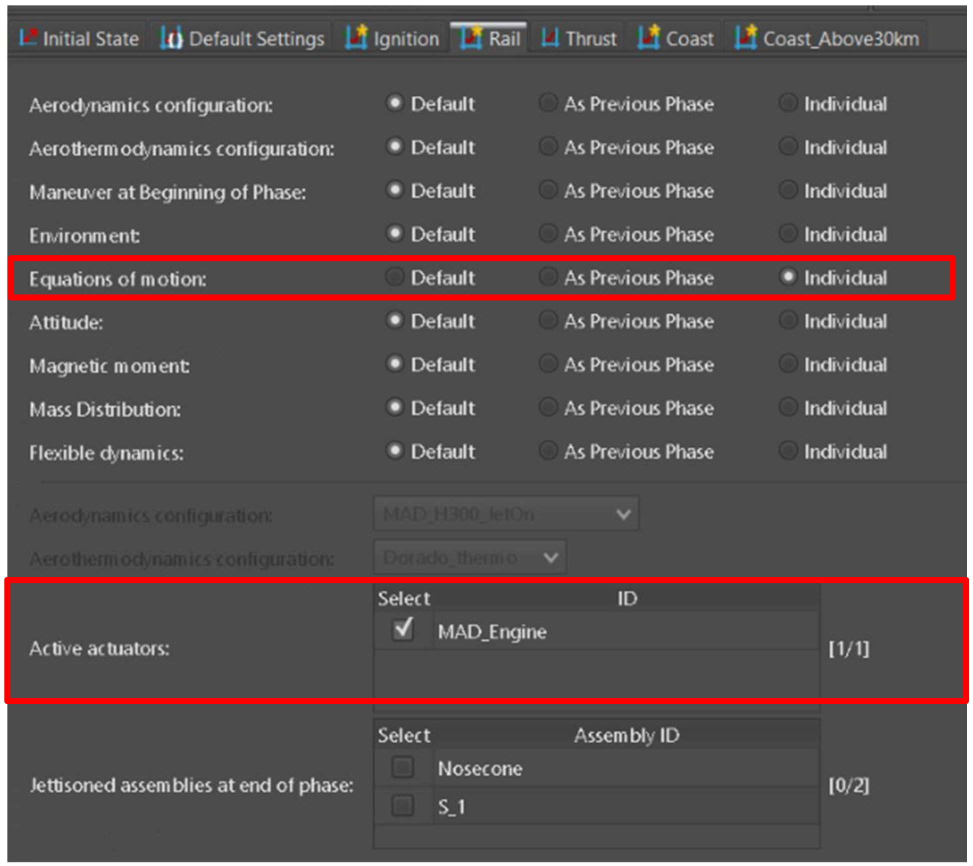
State Variable	Definition
R	Radial distance from planet center
λ	East longitude / Angle between the Greenwich meridian and the meridian of the current position (positive east of Greenwich)
δ	Declination angle between the equatorial plane and the current position vector (positive on the northern hemisphere)

The kinematic state equations represent the kinematic relationship established by the definition of the position:

$$\frac{d}{dt}\begin{bmatrix} R \\ \lambda \\ \delta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.65}$$

Trajectory Parameters

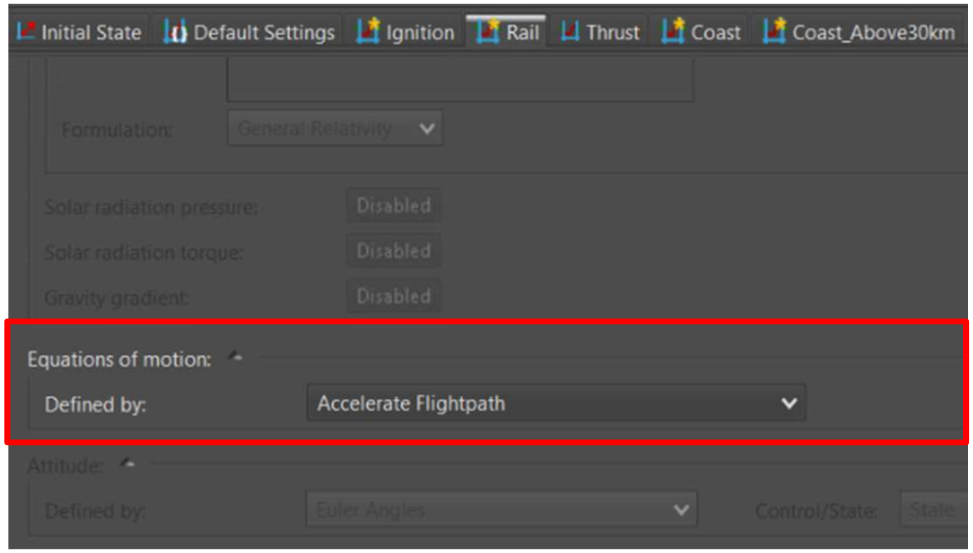
Dynamics - Rail



Active motor

Trajectory Parameters

Dynamics - Rail



Trajectory Parameters

Equations of Motion: Accelerate Flight Path (Astos Model Reference)

Background

The Cartesian components of the velocity vector along the local *L*-frame are given by:

$$\hat{V}_k = V\begin{bmatrix} \sin\gamma \\ \cos\gamma\sin\chi \\ \cos\gamma\cos\chi \end{bmatrix}_L \tag{4.31}$$

The kinematic state equations represent the kinematic relationship established by the definition of the position and the velocity states:

$$\frac{d}{dt}\begin{bmatrix} R \\ \lambda \\ \delta \end{bmatrix} = \begin{bmatrix} V\sin\gamma \\ \frac{V\cos\gamma\sin\chi}{R\cos\delta} \\ \frac{V\cos\gamma\cos\chi}{R} \end{bmatrix} \tag{4.32}$$

The dynamic state equations follow Newton's second law

$$\frac{dV}{dt} = X + \Omega_E^2 R\cos\delta(\cos\delta\sin\gamma - \sin\delta\cos\gamma\cos\chi) \tag{4.33}$$

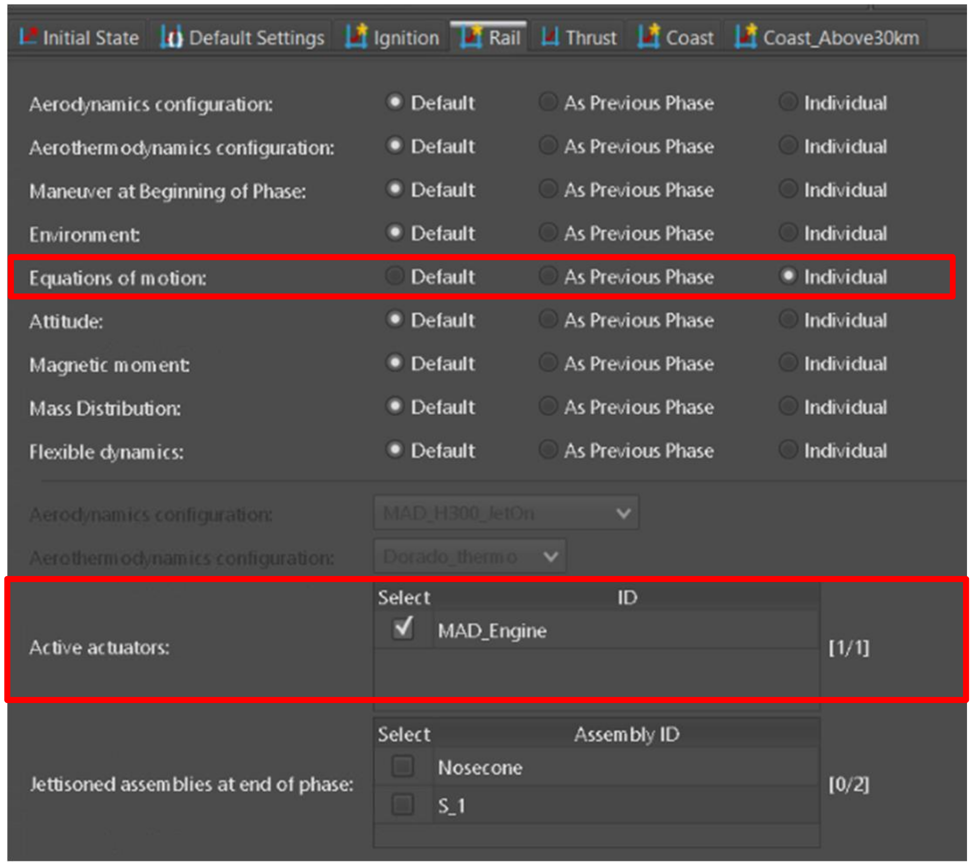
where Ω_E is the angular velocity of the central body about the inertial planet-centered z-axis. *X*, *Y*, *Z* are the components of the acceleration vector in the trajectory coordinate system (see Section 9.3.5.1):

$$\frac{\hat{F}}{m} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_T \tag{4.34}$$

The acceleration acting on the vehicle results from gravity, aerodynamic forces, thrust or other perturbations. Note that only *X* is considered in the dynamic, whereas *Y* and *Z* are neglected (i.e. supposed to be balanced by the rail structure).

Trajectory Parameters

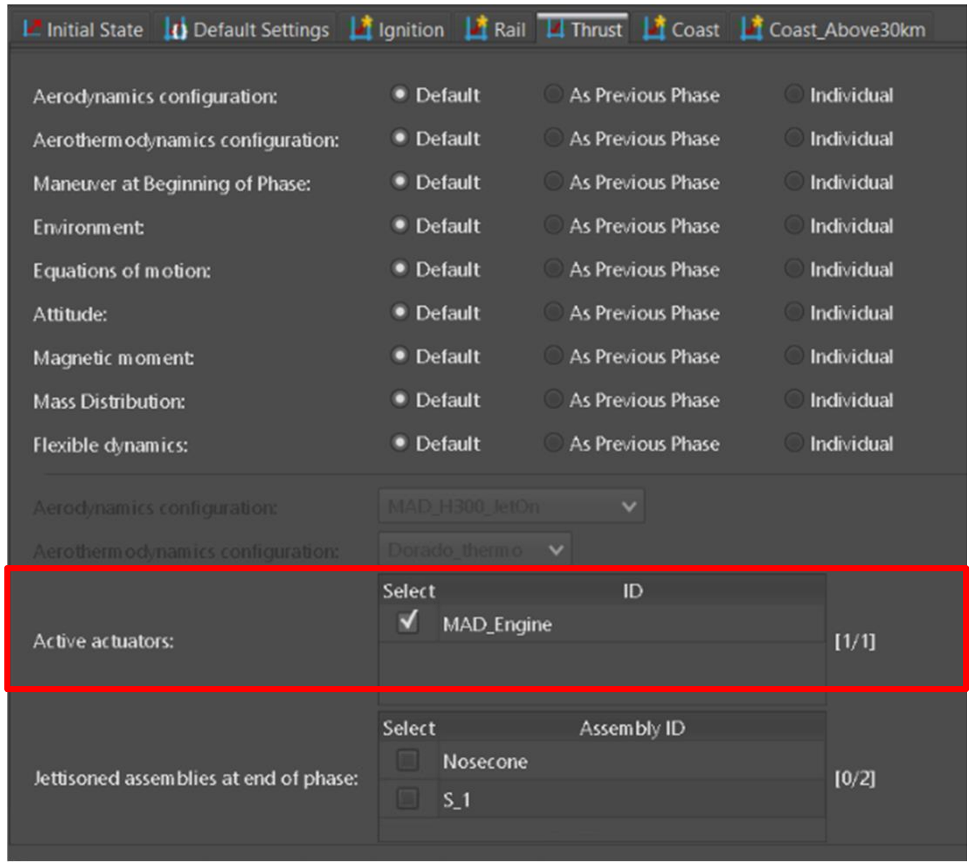
Dynamics - Rail



Active motor

Trajectory Parameters

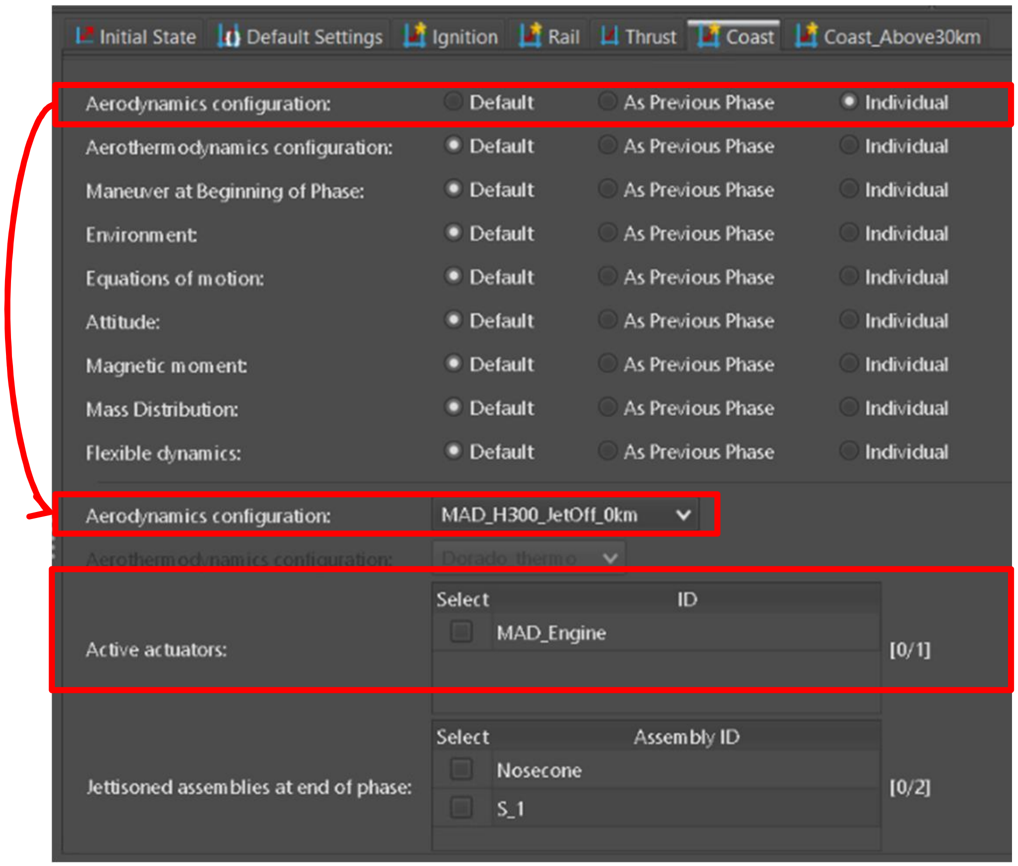
Dynamics - Thrust



Active motor

Trajectory Parameters

Dynamics - Coast

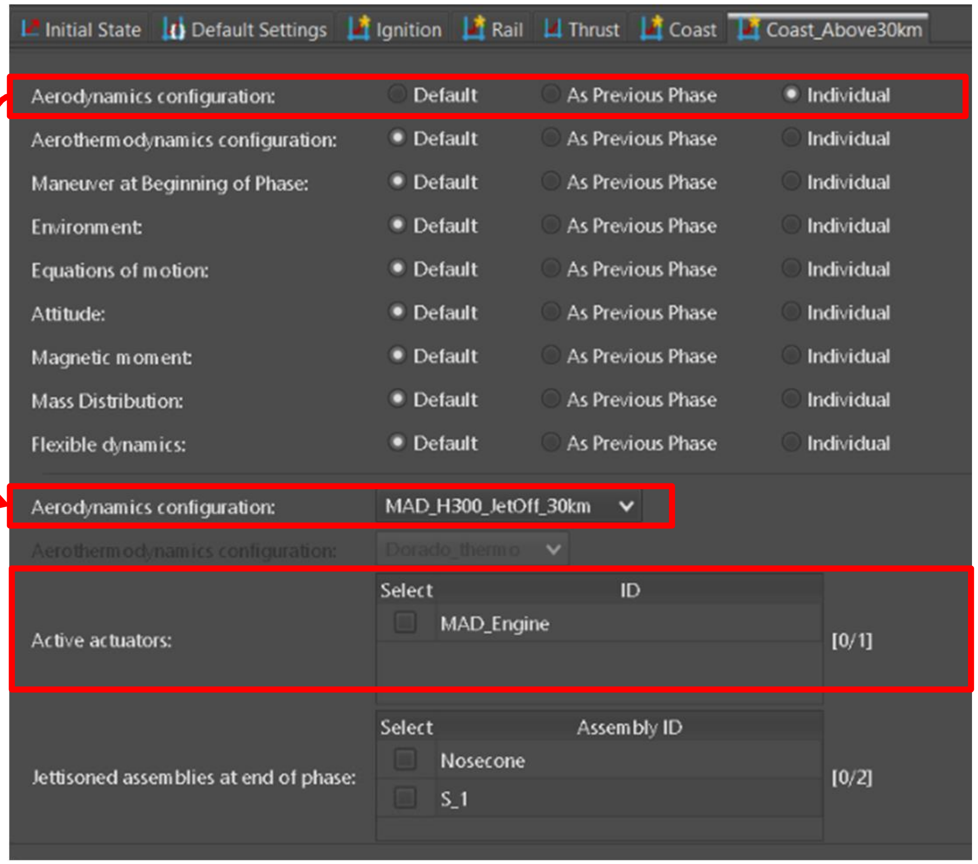


Full base drag – 0 km altitude

Inactive motor

Trajectory Parameters

Dynamics – 2nd Coast Phase



Full base drag – 30 km altitude

Inactive motor

Modelling Guidelines

- **Focus on a single phase at a time**
  - **Disable later phases in the "Phases & Common Settings"**

- **Use curve plots in the "Results" panel for verifying correctness of input/output data**

- **Large aerodynamic tables (i.e. 2D) can be split into series of 1D tables for simulation**
  - **Consequently, the same increase in the no. of trajectory phases are required since each phase utilizes a different aerodynamic table**